Coastal Engineering, 3 (1979) 1—32

EVALUATION OF SEDIMENT TRANSPORT FORMULAE IN COASTAL ENGINEERING PRACTICE

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(Received December 8, 1978; accepted March 10, 1979)

ABSTRACT

Van de Graaff, J. and Van Overeem, J., 1979. Evaluation of sediment transport formulae in coastal engineering practice. Coastal Eng., 3: 1—32.

The CERC formula can only be applied when fairly simple boundary conditions are satisfied. In practice, however, these conditions are often complicated and consequently other sediment transport formulae should be used. Due to the lack of reliable transport measurements in a vertical, a check at prototype conditions of such formulae is almost impossible at the moment and consequently the practical value of some proposed formulae is rather obscure. As a method of verification for the time being, comparative computations have been carried out with the CERC formula and the Bijker formula, the adapted Engelund-Hansen formula and the adapted (2 methods of adaptation) Ackers-White formula. As a result of this study the Bijker formula appeared to be better than the others.

1. INTRODUCTION

Since sediment transports play an important role in various coastal engineering problems, there seems to be an urgent need for a reliable sediment transport formula. To be reliable such a formula should include the effects due to bottom, current and wave conditions. The total longshore sediment transport as a result of longshore currents introduced by oblique wave approach can, for instance, quite simply be computed with the well-known CERC formula. However, tidal and wind-induced currents are generally also important and sometimes the distribution of the sediment transport over the cross-section may also be wanted. For these problems another method of calculation should be used.

In the past various formulae, all of them including the most important parameters encountered in coastal engineering practice, have been proposed (Bijker, 1971; Swart and Delft Hydraulics Laboratory, 1976; Swart, 1976). The use of such formulae is still limited since at present a true verification is almost impossible; the lack of reliable prototype sediment transport measurements being the main reason. Secondly, the multitude of parameters, which should be known before a computation can be executed, adds more difficulties to the problem. Since it will be a long time before sufficient prototype transport measurements in a vertical (especially within the surf zone) are available for a true verification, an alternate method has been proposed. In this method the results obtained with the CERC formula are compared with the results of the proposed sediment transport formulae. Based on various prototype measurements which support the CERC formula, it is assumed that this formula gives a fairly reliable prediction of sediment transport for the relatively simple case of oblique wave approach.

Before such a comparison can be made, the velocities parallel to the coast must be computed. In some degree this seems to be shifting the problems since the computation of longshore currents is complicated and no unanimity exists on this point. Nevertheless, the results of such a comparison are rather clear in the rejection of some proposed sediment transport calculation methods.

Since the velocity distribution perpendicular to the coast to a large extent determines the rate of sediment transport, it seems useful to elaborate on this part extensively (see section 2). Section 3 will deal with the various proposed transport formulae. In section 4 the main results of the computations will be described. A discussion of the results of the comparison is given in section 5. Some conclusions are drawn in section 6.

The computations as described in this paper were originally executed by the Coastal Engineering Group of the Delft University of Technology during the preparation of the Lecture Notes of this Group. Later on the work has been incorporated in the investigations of the Coastal Sediment Group of the Dutch Applied Coastal Research Programme.

2. VELOCITY DISTRIBUTION

The adopted coordinate system is shown in Fig. 1. A straight coastline, parallel depth contours and a constant slope with respect to the still water level will be assumed throughout this paper. Waves approach the coast obliquely and are assumed to be constant along the whole coast. In principle the basic ideas of Bowen (1969) and Longuet-Higgins (1970) are observed in the derivation of the velocity distribution in this section.

In the generation of the longshore current the following classes of forces are important, viz.: radiation stresses; bottom friction and lateral forces.

Under permanent conditions everywhere in the region near the coast, an equilibrium of forces can be expected. In the next paragraphs the various forces, under regular wave condition, will be briefly treated. Then the consequences of an irregular wave field will be described. In the last paragraph of this section some resulting velocity distributions perpendicular to the coast will be given.

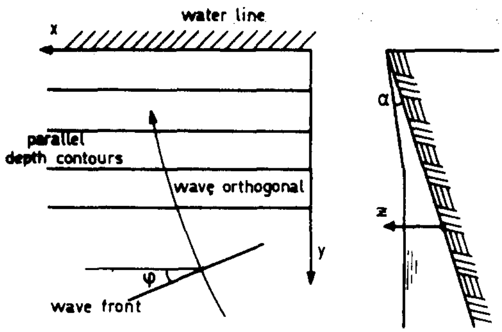

Fig. 1. Definition sketch.

2.1 Radiation stresses

In the chosen situation the radiation shear stress component S_{yx} is the main driving force of the generated longshore currents. Owing to the assumed constant conditions in the x-direction of profiles and waves, the radiation stress component S_{xx} and the wave set-up are constant along the x-axis; therefore they do not contribute to driving forces in the x-direction. Adopting the linear wave theory the shear stress component can be described as:

$$S_{yx} = E\, n \sin\phi \cos\phi \tag{2.1}$$

where: S_{yx} = radiation stress component; n = ratio of group velocity to wave celerity:

$$n = \frac{c_g}{c} \tag{2.2}$$

E = wave energy density; ϕ = angle of wave incidence; c_g = wave group velocity; c = wave celerity. (See also the Appendix.)

Outside the surf zone, conservation of energy flux can be assumed; thus the component S_{yx} is constant there. Inside the surf zone wave breaking causes dissipation of energy and hence the component S_{yx} will decrease towards the coast. Adopting the familiar expressions:

$$E = \frac{1}{8}\rho g H^2 \tag{2.3}$$

$$H = \gamma h \quad \text{(spilling breakers)} \tag{2.4}$$

and:

$$\frac{\sin\phi}{c} = \frac{\sin\phi_0}{c_0} \tag{2.5}$$

eq. 2.1 can be rewritten as:

$$S_{yx} = \frac{1}{8}\rho g H^2\, n \frac{c}{c_0} \sin\phi_0 \cos\phi \tag{2.6a}$$

or within the surf zone:

$$S_{yx} = \frac{1}{8}\rho g\gamma^2 h^2\, n\, \frac{c}{c_0}\, \sin\phi_0 \cos\phi \qquad (2.6b)$$

where: ρ = mass density of water; g = gravitational acceleration; H = local wave height; γ = wave braking index; h = local water depth (including wave set-up). Subscript "o" denotes deep-water conditions.

The component S_{yx} is a function of y since h, n, c and ϕ are in principle functions of y.

On a vertical column of water with unit length in the x and y direction, a resulting radiation shear stress acts:

$$\frac{dS_{yx}}{dy} = \frac{d}{dy}\left(\frac{1}{8}\rho g\gamma^2 h^2\, n\, \frac{c}{c_0}\, \sin\phi_0 \cos\phi\right) \qquad (2.7)$$

With the familiar approximations within the surf zone, viz.:

$$n = 1 \qquad (2.8)$$

$$c = (gh)^{\frac{1}{2}} \qquad (2.9)$$

$$\cos\phi = 1 \qquad (2.10)$$

eq. 2.7 can be rewritten as:

$$\frac{dS_{yx}}{dy} = \frac{5}{16}\rho\gamma^2 (gh)^{3/2}\, \frac{\sin\phi_0}{c_0}\, \mathrm{tg}\alpha \qquad (2.11)$$

where $\mathrm{tg}\alpha = dh/dy$ (the slope of the profile with respect to the actual mean water surface).

2.2 Bottom friction

The friction at the bottom plane of the water column counteracts the driving forces. The bottom friction belonging to a longshore current generated by oblique waves is an example of the complex interaction of shear stresses caused by waves and currents. In this paper the original approach of Bijker (1967) will be applied with some adjustments as suggested by Swart (1974) to the work of Jonsson (1966).

The combined action of waves and current has an extra effect on the bottom friction in the direction of the current when compared to the bottom friction by current alone. The mean value averaged over a wave period can be computed from an elliptic integral which Bijker (1967) approximates with the formula:

$$\tau'_{wc} = \tau_c \left[a + b\left(\xi \frac{\hat{u}_0}{v}\right)^c\right] \qquad (2.12)$$

where τ'_{wc} = mean component of the bottom shear stress in the direction of the current as a result of waves and current; τ_c = bottom shear stress due to the same current without waves; ξ = parameter:

$$\xi = C(f_w/2g)^{\frac{1}{2}} \tag{2.13a}$$

(where C = Chézy coefficient, and f_w = bottom friction coefficient according to Jonsson, 1966); $\hat{u}_0$ = amplitude of the orbital velocity at bed; v = mean current velocity; a, b, c = parameters depending on the angle between current and waves. Since the current is parallel to the coast line, the angle can be indicated by ϕ.

Bijker (1967) originally determined his parameter ξ:

$$\xi = p\,\kappa\,C\,g^{-\frac{1}{2}} \tag{2.13b}$$

where κ = Von Kármán coefficient; p = constant (model tests have indicated: $p \simeq 0.45$). This parameter ξ followed from his assumption of the wave-induced bottom shear stress yielding:

$$\tau_w = \rho\kappa^2 p^2\, u_0 \left| u_0 \right| \tag{2.14a}$$

Jonsson (1966), however, stated:

$$\tau_w = \tfrac{1}{2}\rho f_w\, u_0 \left| u_0 \right| \qquad (f_w \text{ a function of } a_0/r) \tag{2.14b}$$

where τ_w = bottom shear stress due to waves; a_0 = amplitude of orbital excursion at bed; r = bed roughness.

Comparison of the results of Bijker and Jonsson yields:

$$p = \frac{1}{\kappa}(f_w/2)^{\frac{1}{2}} \tag{2.15}$$

Since f_w is variable (dependent upon a_0 and r), p should also vary. If eq. 2.15 is introduced in the original Bijker parameter eq. 2.13b, eq. 2.13a results.

Due to refraction the angle ϕ is small in and near the surf zone. Generally ϕ will be less than 20°. For $0° < \phi < 20°$ eq. 2.12 results in (see Bijker, 1967):

$$\tau'_{wc} = \tau_c \left[.75 + .45 \left(\xi \frac{\hat{u}_0}{v} \right)^{1.13} \right] \tag{2.16}$$

The approximative formula 2.16 holds for $\xi(\hat{u}_0/v) > 1$.

Starting from the original formula, within the surf zone a further approximated formula can be derived. Incorporating $\phi \simeq 0$ and $\xi(\hat{u}_0/v) >> 1$ the ultimate result becomes:

$$\tau'_{wc} = \tau_c \frac{2}{\pi} \xi \frac{\hat{u}_0}{v} \tag{2.17}$$

Inserting eq. 2.13a for ξ and $\tau_c = \rho g(v^2/C^2)$, formula 2.17 results in:

$$\tau'_{wc} \simeq \frac{\sqrt{2}}{\pi} \rho g^{\frac{1}{2}} \frac{f_w^{\frac{1}{2}}}{C} v\hat{u}_0 \tag{2.18a}$$

When instead of the Chézy coefficient, the Darcy-Weisbach coefficient ($f = 8g/C^2$) is taken into eq. 2.18a, a more "regular" formula results:

$$\tau'_{wc} \simeq \frac{\rho}{2\pi} (f\, f_w)^{\frac{1}{2}}\, v\hat{u}_0 \tag{2.18b}$$

In eq. 2.18a aspects of the waves ($\hat{u}_0$ and f_w) and of the current (v and C) are present. In the literature other friction terms are frequently encountered. Bowen (1969) gives a friction term with only current-dependent parameters. Longuet-Higgins (1972) introduces a bottom-friction formula with the product of $\hat{u}_0$ and v and as a friction coefficient only f_w. Jonsson et al. (1974) propose a resulting friction factor which incorporates effects of waves and current. In the present case (weak current in comparison to the orbital velocity) they find a friction factor with only f_w and the product of $\hat{u}_0$ and v.

With $\hat{u}_0 \simeq \frac{1}{2}\gamma\, (gh)^{\frac{1}{2}}$, formula (2.18a) can be simplified to:

$$\tau'_{wc} \simeq \frac{1}{\pi\sqrt{2}}\, \rho g \gamma\, \frac{f_w^{\frac{1}{2}}}{C}\, v h^{\frac{1}{2}} \tag{2.19}$$

With the eqs. 2.11 and 2.19 and neglecting lateral forces, a simple direct expression of v as a function of the water depth h can be found. In computer applications the more complicated eqs. 2.7 and 2.16 can better be used (see paragraph 2.5).

2.3 Lateral forces

The driving forces for the longshore current, which are connected with the radiation shear stress, are only present within the surf zone. Due to lateral forces a redistribution of the longshore currents results. Principally this lateral force can be described by:

$$\tau_l = \rho \epsilon h \frac{dv}{dy} \tag{2.20}$$

where τ_l = lateral friction force; ϵ = turbulent diffusion coefficient; dv/dy = gradient of longshore current velocity. Problems arise in quantifying ϵ. Longuet-Higgins (1970) for instance, suggests ϵ to be proportional to a characteristic mixing length L and a characteristic turbulent velocity U.

$\epsilon :: L\, U$

The estimations of Longuet-Higgins for L and U lead to a wide range of possible values of ϵ as a function of depth. Finally results:

$$\epsilon = N_L\, y\, (gh)^{\frac{1}{2}} \tag{2.21a}$$

with N_L a constant of proportionality; $0 < N_L < 0.02\gamma$.

Battjes (1975) postulates that the dissipation of wave energy is the main source of turbulent velocity fluctuations. Moreover, the local depth is taken as a characteristic length. For plane slopes the ultimate result, written in a similar form as eq. 2.21a, yields:

$$\epsilon = N_B \, y \, (gh)^{\frac{1}{2}} \tag{2.21b}$$

In this model N_B is a function of the wave-breaking index and the bottom slope:

$$N_B = M \left(\frac{5\gamma^2}{16}\right)^{1/3} (\text{tg}\alpha)^{4/3} \tag{2.22}$$

where M is a constant of order 1.

In prototype conditions the value of N_B is generally in the lowermost part of the N_L-domain as given by Longuet-Higgins.

Other estimates of ϵ are given for example by Bowen (1969), Thornton (1970) and Jonsson et al. (1974).

2.4 Irregular waves

In the preceding paragraphs regular waves have been assumed. Generally, however, in prototype irregular waves are present. The greater part of the indicated forces acts within the surf zone and they turn out to be functions of local wave parameters. However, no conclusive, verified description of local wave parameters in the surf zone due to an irregular wave field is yet available. Battjes (1974) described the behaviour of an irregular wave field with a constant period and constant direction. He introduces fictitious wave heights H_f which can be calculated from shoaling and refraction. He obtains:

$$\overline{H_f^2} = \frac{1}{2n} \frac{c_0}{c} \frac{\cos\phi_0}{\cos\phi} \overline{H_0^2} \tag{2.23}$$

where $\overline{H_f^2}$ = mean square of fictitious wave heights; $\overline{H_0^2}$ = mean square of deep water wave heights.

Far outside the surf zone these fictitious wave heights are in fact real; inside the surf zone they are indeed partly fictitious. Battjes assumes that H_f is Rayleigh-distributed and states that the local wave height does not exceed γh; all originally higher fictitious waves are reduced to γh.

The shear stress component of the radiation stress, eq. 2.6a, includes $\overline{H^2}$. With irregular waves it seems logical to insert $\overline{H^2}$ to get the mean value with respect to time of this component. The next expression can be derived:

$$\overline{H^2} = [1 - \exp(-\gamma^2 h^2/\overline{H_f^2})] \, \overline{H_f^2} \tag{2.24}$$

In the bottom-friction expression 2.16, $(\hat{u}_0)^{1.13}$ appears; via f_w there is also some effect of $\hat{u}_0$ on the ultimate results. The maximum orbital velocity near the bottom $\hat{u}_0$ is, according to the linear wave theory, proportional to H. Calculations of some plausible prototype conditions lead to the admissibility to take $\overline{H}$ as a characteristic wave height for the mean bottom friction due to an irregular wave field (see Fig. 2A). $\overline{H}$ can be computed from:

$$\overline{H} = \frac{\pi^{\frac{1}{2}}}{2} \{\overline{H_f^2}\}^{\frac{1}{2}} \operatorname{erf}\,[\gamma h / \{\overline{H_f^2}\}^{\frac{1}{2}}] \qquad (2.25)$$

in which erf is the error function defined by:

$$\operatorname{erf}(a) = \frac{2}{(\pi)^{\frac{1}{2}}} \int_0^a e^{-u^2}\, du \qquad (2.26)$$

As will be described in the next paragraph, no velocity distribution calcula-

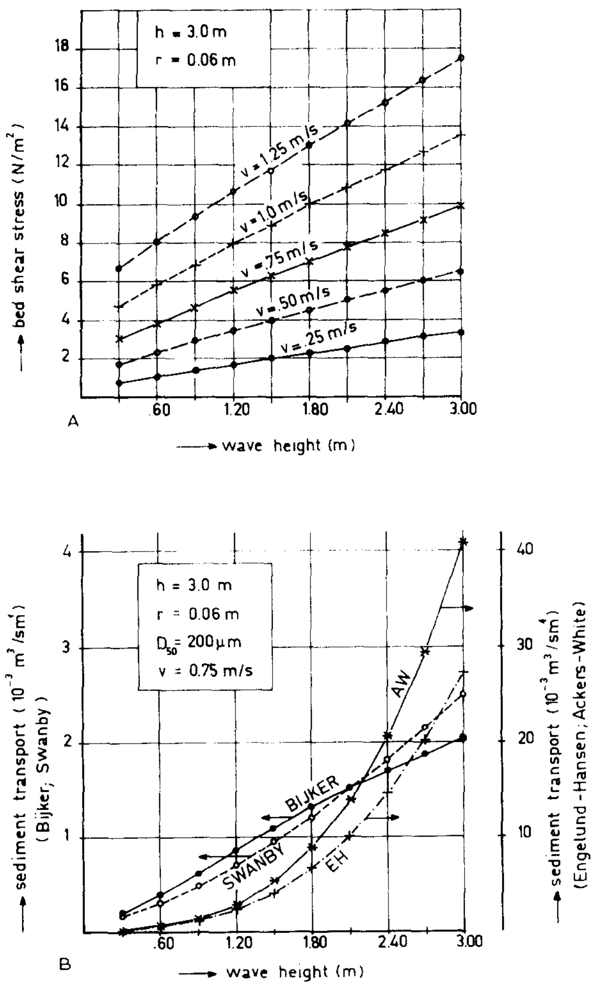

Fig. 2. A. Bed shear stress as a function of the wave height. B. Sediment transport as a function of the wave height.

tions have been carried out with lateral friction effects in an irregular wave field; therefore a consideration of the effect of random waves upon the lateral friction will be omitted.

The description of the effect of irregular waves on the radiation shear stress and on the bottom shear stress as given in this paragraph, started from the description of the local wave height within the surf zone. Battjes and Janssen (1978) developed an alternative model in which the energy dissipation is estimated from the energy dissipation in a bore. This energy dissipation can be used directly to compute the radiation shear stress component S_{yx}. This alternative model can, contrary to the model as used in this paper, easily be applied in a bar-trough-beach profile.

2.5 *Resulting velocity distribution*

In a steady state, equilibrium of forces results in:

$$\frac{dS_{yx}}{dy} + \frac{d\tau_l}{dy} - \tau'_{wc} = 0 \tag{2.27}$$

To find some different velocity distributions various combinations of forces as described in the preceding paragraphs have been introduced (see Fig. 3). In all cases the same wave conditions have been applied viz.: wave height in deep water $H_0 = 2$ m (when random waves are used $H_{0rms} = 2$ m); wave period $T = 7$ sec; angle of approach in deep water $\phi_0 = 30°$ ($\phi_{br} \simeq 13°$); wave breaking index $\gamma = 0.8$; slope of profile with respect to water level tgα = 1:100; bed roughness (constant over whole profile) $r = 0.06$ m.

Fig. 3, distribution a. Regular waves; no lateral friction ($\tau_l = 0$). The "approximated" formulae 2.11 and 2.19 have been used as radiation stress component and bottom friction expression. The velocity distribution becomes:

$$v = \frac{5\pi}{8\sqrt{2}} \gamma h \frac{g^{\frac{1}{2}}C}{f_w^{\frac{1}{2}}} \frac{\sin\phi_0}{c_0} \operatorname{tg}\alpha \tag{2.28}$$

A nearly linear distribution results as a function of distance from the waterline. The divergences from the linear relation are due to the variation of the factor $C/f_w^{\frac{1}{2}}$ with h.

Fig. 3, distribution b. Regular waves; no lateral friction ($\tau_l = 0$). The "less approximated' formulae 2.7 and 2.16 have been applied. With the chosen boundary conditions the use of the approximations from distribution a, results obviously in an increase of velocity of about 25%.

Fig. 3, distribution c. Regular waves; lateral friction according to Longuet-Higgins with the maximum lateral mixing effect ($N_L = 0.02\gamma = 0.016$). To get an analytical solution of eq. 2.27 the "approximated" eqs. 2.11 and 2.19 should be used. Moreover, a constant friction term is introduced in eq. 2.19; the breaker line conditions are applied for $f_w^{\frac{1}{2}}/C$. The relatively strong effect of the lateral friction results in a smoothed velocity distribution curve.

Fig. 3, distribution d. Regular waves; lateral friction according to Battjes. In this case $N_B = M\left(\frac{5}{16}\gamma^2\right)^{1/3}(\text{tg}\alpha)^{4/3} = .0013$ ($M = 1$). A less smoothed curve is obtained in comparison with distribution c.

Fig. 3, distribution e. Irregular waves; no lateral friction ($\tau_l = 0$). In principle the "less approximated" formulae 2.7 and 2.16 have been applied together with the adjustments as described in paragraph 2.4. Due to the successive breaking of the individual waves of the irregular wave field and hence the gradual production of the driving force, a very smoothed velocity distribution curve results. With this gradual distribution the lateral forces will be relatively small and together with the uncertainties with respect to the use of the correct diffusion coefficient, it seems less necessary to introduce lateral

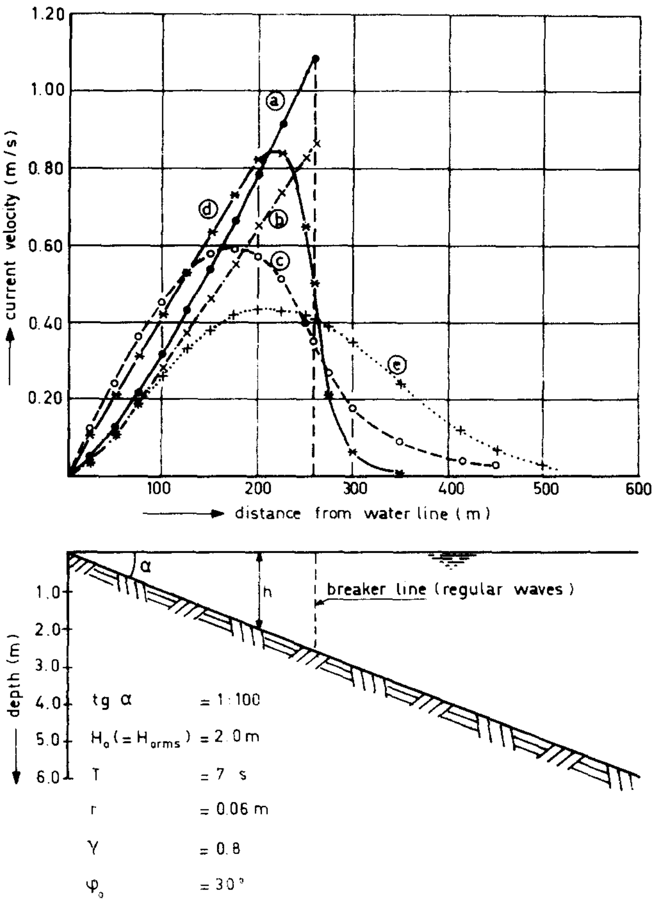

Fig. 3. Example velocity distribution profiles.

friction forces in order to adjust the velocity distribution further. These are the reasons why lateral friction forces have been excluded when an irregular wave field is considered. In the calculations as described in section 4 irregular waves have been applied.

3. SEDIMENT TRANSPORT FORMULAE

3.1 CERC formula

The intention of this study is to compare the results of various sediment transport formulae with the results of the CERC formula. In the assumed beach configuration the total sediment transport along the beach can be expressed as:

$$S_{\mathrm{CERC}} = A\, H_0^2\, c_0 \sin\phi_{\mathrm{br}} \cos\phi_0 \qquad (3.1)$$

in which S_{CERC} = total littoral transport; A = dimensionless coefficient; H_0 = deep water wave height; c_0 = deep water wave celerity; ϕ_{br} = angle of wave incidence at the breaker line; ϕ_0 = idem in deep water.

Notwithstanding the simple character of this formula, some confusion exists with respect to the choice of the wave height and the dimensionless coefficient A.

Due to the term H_0^2 in eq. 3.1, the application of $H_{0\mathrm{rms}}$ should be preferable in an irregular wave field. However, in most cases $H_{0\,\mathrm{sign}}$ is used. In a narrow-banded Rayleigh-distributed wave field holds:

$$(H_{\mathrm{sign}})^2 \simeq 2(H_{\mathrm{rms}})^2 \qquad (3.2)$$

In applying H_{rms} in formula 3.1 the coefficient A should have twice the value of coefficient A attributed when using H_{sign}.

Based on prototype and model results in the past, the coefficient A has been evaluated as 0.014 (using H_{sign}). Studies by, among others, Komar (1971) point to higher values of A. The Shore Protection Manual recommends at the moment (after conversion of units) a value of 0.025 (again using H_{sign}). Galvin and Vitale (1976) motivate this increase of A and hence rise of transport (by 83%!).

In Fig. 4 the measured sediment transport of 32 prototype cases have been given as a function of the parameter ($H_{0\,\mathrm{rms}}^2\ c_0 \sin\phi_{\mathrm{br}}\cos\phi_0$). A linear scale has been used. When the exceptionally situated point of Moore and Cole is excluded, and a (linear) least-squares fit is used, a coefficient A of 0.036 results. (In this fitting it is assumed that, just as in the CERC formula, a straight line through the origin should result.)

If the coefficient A is computed for every separate measuring point and the average value is taken, A = 0.047 results (Moore and Cole excluded). It is not clear which method of computing A should be preferred. In the calculations of section 4 the mean value of the two methods has been adopted. This results in A = 0.042. This value is slightly less than the value as recommended in the Shore Protection Manual. [$A = (H_{\mathrm{sign}}/H_{\mathrm{rms}})^2 \times 0.025 \simeq 2 \times 0.025 \simeq 0.050$]

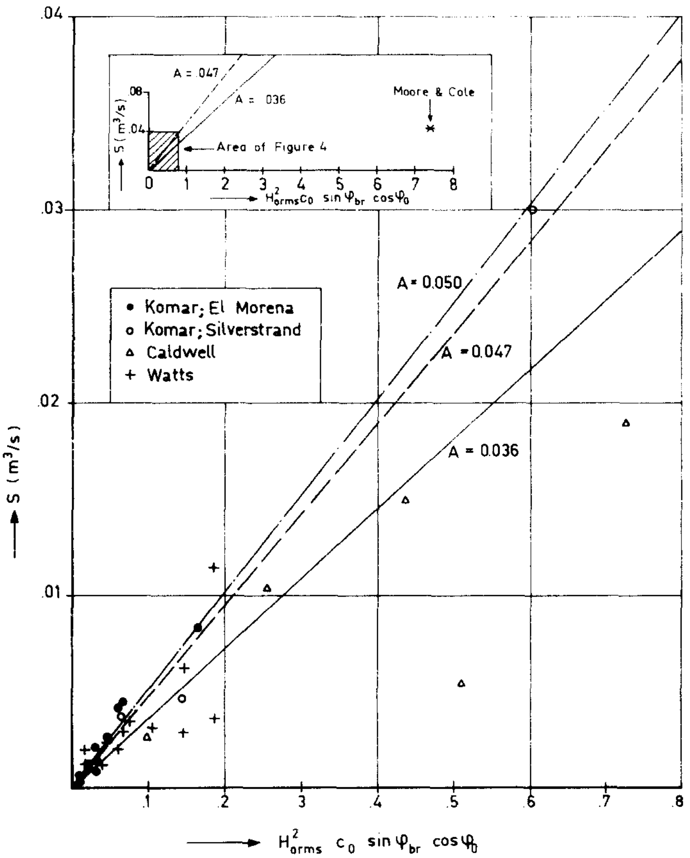

Fig. 4. Longshore sediment transport prediction.

3.2 Other sediment transport formulae

Most of the sediment transport formulae used in coastal engineering practice have been derived from formulae as used in river sediment computations. Apparently such formulae can only be used when the effect of the waves is properly included.

In almost every sediment transport formula the total transport S is described as:

$$S = (v) * (\text{“load”}) \tag{3.3}$$

in which the “load”-parameter is connected with the amount of rolling, saltating and suspended particles. Although normally the mean current velocity is encountered in the “load”-parameter, it seems physically better to use the bottom shear stress τ_c instead. In mere current, v and τ_c are indeed exchangeable via:

$$\tau_c = \rho g \frac{v^2}{C^2} \tag{3.4}$$

Bijker (1967) examined the increasing effect of waves on the bottom friction in comparison with the friction due to current alone. He suggested to apply this increased friction in the "load"-parameter of a river sediment transport formula to obtain a formula which can be applied in coastal engineering transport calculations.

Bijker (1971) worked out his basic idea on the Kalinske-Frijlink formula, which predicts the transport near the bottom, in combination with Einstein's conception for the prediction of the suspended load (Einstein, 1950; Frijlink, 1952). This combination will be called the Bijker formula. At the moment newer river sediment transport formulae are available (Ackers and White, 1973; Engelund and Hansen, 1967). Under the auspices of the Coastal Sediment Group of the Dutch Applied Coastal Research Programme, Swart and Delft Hydraulics Laboratory (1976), using methods comparable with Bijker's, adapted the latter formulae to the combination of current and waves as boundary conditions. (See also Swart, 1976.)

Since details on the applied formulae can be found in the original papers, in the Swart and Delft Hydraulics Laboratory Report (1976) and in the Swart (1976) paper, the formulae are only briefly described in this paper.

Bijker formula

Starting from the Kalinske-Frijlink formula, Bijker (1967) suggests the following formula for the bed load in the coastal environment:

$$S_b = 5\, D_{50} \frac{v}{C} g^{\frac{1}{2}} \exp \left[\frac{-0.27\, \Delta D_{50} \rho g}{\mu \tau_c \left[1 + \frac{1}{2} \left(\xi \frac{\hat{u}_0}{v} \right)^2 \right]} \right] \tag{3.5}$$

where S_b = bed load (in m^3/sm including pores); D_{50} = particle diameter (50% by weight exceeded in size); v = mean current velocity; C = Chézy coefficient; g = gravitational acceleration; Δ = relative apparent density of bed material:

$$\Delta = \frac{\rho_s - \rho}{\rho} \tag{3.6}$$

(in which ρ_s = mass density of bed material); ρ = mass density of water; μ = ripple factor:

$$\mu = \left(\frac{C}{C_{90}} \right)^{1.5} \tag{3.7}$$

C_{90} = Chézy coefficient based on D_{90} ; D_{90} = particle diameter (10% by weight exceeded in size); ξ = Bijker's parameter:

$$\xi = C\, (f_w / 2g)^{\frac{1}{2}} \tag{3.8}$$

(in which f_w = Jonsson's friction factor); τ_c = bottom shear stress due to current; $\hat{u}_0$ = maximum orbital velocity at bed.

The bed load is assumed to take place in a layer with thickness r (= bed roughness) above the bed. The suspended load becomes:

$$S_{sus} = 1.83\, S_b \left[I_1 \ln\left(\frac{33h}{r}\right) + I_2 \right] \tag{3.9}$$

in which S_{sus} = suspended load (in part of vertical above $z = r$); h = local water depth; r = bed roughness; I_1, I_2 = Einstein's integrals:

$$I_1 = \frac{.216\left(\frac{r}{h}\right)^{(z_*-1)}}{\left(1-\frac{r}{h}\right)^{z_*}} \int_{(r/h)}^{1} \left(\frac{1-y}{y}\right)^{z_*} dy \tag{3.10}$$

$$I_2 = \frac{.216\left(\frac{r}{h}\right)^{(z_*-1)}}{\left(1-\frac{r}{h}\right)^{z_*}} \int_{(r/h)}^{1} \left(\frac{1-y}{y}\right)^{z_*} \ln y\, dy \tag{3.11}$$

$$z_* = \frac{w}{\kappa\, v_{*wc}} \tag{3.12}$$

w = sediment particle fall velocity; κ = Von Kármán coefficient;

$$v_{*wc} = v_{*c} \left[1 + \tfrac{1}{2}\left(\xi \frac{\hat{u}_0}{v}\right)^2\right]^{\frac{1}{2}} \tag{3.13}$$

v_{*c} = shear stress velocity due to current.

The total sediment transport yields:

$$S_{Bijker} = S_b + S_{sus} \tag{3.14}$$

Adapted Engelund-Hansen formula

Expressed as $S = (v) * $ ("load") the Engelund-Hansen formula can be rewritter as:

$$S_{EH} = v \frac{.05\, C\, \tau_c^2}{\rho^2 g^{5/2} \Delta^2 D_{50}} \tag{3.15}$$

where S_{EH} = total sediment transport; v = mean current velocity; C = Chézy coefficient; τ_c = bottom shear stress due to current; ρ = mass density of water; g = gravitational acceleration; Δ = relative apparent density of bed material; D_{50} = particle diameter (50% by weight exceeded in size).

With the increased bottom shear stress due to waves:

$$\tau_{wc} = \tau_c \left[1 + \tfrac{1}{2}\left(\xi \frac{\hat{u}_0}{v}\right)^2\right] \tag{3.16}$$

formula 3.15 yields

$$S_{\mathrm{EH}} = v \frac{.05\, C\, \tau_{\mathrm{c}}^{2} \left[1 + \frac{1}{2}\left(\xi \frac{\hat{u}_0}{v}\right)^2\right]^2}{\rho^2 g^{5/2} \Delta^2 D_{50}} \tag{3.17}$$

Adapted Ackers-White formula

The original Ackers and White formula, expressed in the present symbols, yields:

$$S_{\mathrm{AW}} = v \frac{1}{(1-p)} D_{35} \left(\frac{v}{v_{*\mathrm{c}}}\right)^n \frac{C_{\mathrm{D_{gr}}}}{A^m} (F_{\mathrm{c}} - A)^m \tag{3.18}$$

where S_{AW} = total sediment transport; p = porosity of sedimented material; v = mean current velocity; D_{35} = particle diameter (65% by weight exceeded in size); $v_{*\mathrm{c}}$ = shear stress velocity due to current; n = dimensionless parameter:

$$n = 1 - 0.2432 \ln(D_{\mathrm{gr}}) \tag{3.19}$$

$$D_{\mathrm{gr}} = D_{35} \left(\frac{g\Delta}{\nu^2}\right)^{1/3} \tag{3.20}$$

Δ = relative apparent density of bed material; ν = kinematic viscosity of water; g = gravitational acceleration; $C_{\mathrm{D_{gr}}}$ = dimensionless parameter:

$$C_{\mathrm{D_{gr}}} = \exp\left[2.86 \ln(D_{\mathrm{gr}}) - 0.4343 [\ln(D_{\mathrm{gr}})]^2 - 8.128\right] \tag{3.21}$$

A = dimensionless parameter:

$$A = \frac{0.23}{D_{\mathrm{gr}}^{\frac{1}{2}}} + 0.14 \tag{3.22}$$

m = dimensionless parameter:

$$m = \frac{9.66}{D_{\mathrm{gr}}} + 1.34 \tag{3.23}$$

F_{c} = sediment mobility number:

$$F_{\mathrm{c}} = \frac{v \left(\frac{v_{*\mathrm{c}}}{v}\right)^n C_{\mathrm{D}}^n}{C_{\mathrm{D}}\, g^{n/2} (\Delta D_{35})^{\frac{1}{2}}} \tag{3.24}$$

$$C_{\mathrm{D}} = 18 \log\left(\frac{10h}{D_{35}}\right) \tag{3.25}$$

h = local water depth.

Ackers and White assume a substantial difference in the mode of transport of coarse and fine grains. They state that the fine sediments travel largely in suspension and that the rate of transport depends on the total shear on the bed. That is why v_{*c} appears in the equations (3.18) and (3.24). The coarse-grain sediment transport is assumed to be dependent on the actual shear stress on the grains. Ackers and White assume that this stress is comparable with the shear stress which would be present in case of a plane granular surface bed with the same mean stream velocity. They express this particular shear stress in terms of v [compare equation (3.4)] and therefore v appears in the equations (3.18) and (3.24). The first v after the equal sign in equation (3.18) is from another source; it indicates the dependence of the transport on the total discharge. Swart and Delft Hydraulics Laboratory (1976) (see also Swart, 1976) assumed that the increasing effect of the waves on the sediment transport rate as computed with the Ackers-White formula can be introduced by replacing v_{*c} in the eqs. 3.18 and 3.24 by v_{*wc} $\left[v_{*wc} = v_{*c} \left[1 + \frac{1}{2}\left(\xi \frac{\hat{u}_0}{v}\right)^2\right]^{\frac{1}{2}} \right]$. This approach has been called the SWANBY method (Swart, 1976). In the sections 4 and 5 this formula will be applied. The ultimate result becomes:

$$S_{\mathrm{SWANBY}} = v \frac{1}{(1-p)} D_{35} \left[\frac{v}{v_{*c}\left[1 + \frac{1}{2}\left(\xi \frac{\hat{u}_0}{v}\right)^2\right]^{\frac{1}{2}}} \right]^n \frac{C_{\mathrm{D_{gr}}}}{A^m} \left[\frac{v \left[\frac{v_{*c}\left[1 + \frac{1}{2}\left(\xi \frac{\hat{u}_0}{v}\right)^2\right]^{\frac{1}{2}}}{v} \right]^n C_{\mathrm{D}}^n}{C_{\mathrm{D}}\, g^{n/2} (\Delta D_{35})^{\frac{1}{2}}} - A \right]^m \tag{3.26}$$

This method of adaptation, however, seems to be incomplete, since it takes no account of the effect of the orbital motion on the actual shear stress on the grains. Extending the original ideas of Ackers and White, the increasing effect of the waves on v should be computed by assuming a flat bed with the bed material diameter as roughness elements. (It will be obvious that in this case v should be considered as a general velocity and should not be considered exclusively as mere current velocity.) The resulting formula is:

$$S_{\mathrm{AW}} = v \frac{1}{(1-p)} D_{35} \left[\frac{v\left[1 + \frac{1}{2}\left(\xi' \frac{\hat{u}_0}{v}\right)^2\right]^{\frac{1}{2}}}{v_{*c}\left[1 + \frac{1}{2}\left(\xi \frac{\hat{u}_0}{v}\right)^2\right]^{\frac{1}{2}}} \right]^n \frac{C_{\mathrm{D_{gr}}}}{A^m} \left[\frac{v\left[1 + \frac{1}{2}\left(\xi' \frac{\hat{u}_0}{v}\right)^2\right]^{\frac{1}{2}} \left[\frac{v_{*c}\left[1 + \frac{1}{2}\left(\xi \frac{\hat{u}_0}{v}\right)^2\right]^{\frac{1}{2}}}{v\left[1 + \frac{1}{2}\left(\xi' \frac{\hat{u}_0}{v}\right)^2\right]^{\frac{1}{2}}} \right]^n C_{\mathrm{D}}^n}{C_{\mathrm{D}}\, g^{n/2} (\Delta D_{35})^{\frac{1}{2}}} - A \right]^m \tag{3.27}$$

where ξ' = Bijker's parameter for shear stress on grains in a flat bed.

$$\xi' = C_D \, (f'_w/2g)^{\frac{1}{2}} \tag{3.28}$$

$$C_D = 18 \log \frac{10h}{D_{35}} \tag{3.29}$$

f'_w = Jonsson's friction factor with D_{35} as bed roughness.
For comparison: ξ = Bijker's parameter:

$$\xi = C \, (f_w/2g)^{\frac{1}{2}} \tag{3.30}$$

$$C = 18 \log \frac{12h}{r} \tag{3.31}$$

f_w = Jonsson's friction factor with r as bed roughness.

In the following chapters the adaptation of the Ackers-White formula according to eq. 3.27 will also be applied; it is called the adapted Ackers-White formula.

4. RESULTS

4.1 Boundary conditions

Various boundary conditions with respect to: wave conditions, H_{0rms} and T, angle of wave incidence, ϕ_0, wave breaking index, γ, diameter of bed material, D_{50}, slope of beach profile with regard to the mean still water level, tgα, bed roughness, r, have been assumed as a basis for comparative calculations. The CERC formula is sensitive to only the first three conditions mentioned. In the CERC formula the value of $\sin\phi_{br}$ belonging to a regular wave field with $H_0 = H_{0rms}$ has been adopted. Sediment transport computations with the four other formulae have been executed after calculation of the velocity distribution in accordance with the method described in section 2 for irregular waves. The boundary conditions mentioned above (except the diameter of the bed material) are important in the resulting velocity distribution. The applied boundary conditions are summarized in Table I.

Bed materials

The used bed materials are characterized by D_{50} in Table I. In the various formulae other quantities are necessary. Table II gives a summing-up of the applied values.

Slope of beach profile

As boundary conditions three slopes have been adopted viz.: tgα = 1:100; 1:50 and 1:20. All cases relate to the beach slope with regard to the actual still-water level. The wave set-up is included in this level. Thus the actual beach slope with respect to the horizontal is slightly steeper.

TABLE I

Boundary conditions

Wave height $H_{0\,\mathrm{rms}}$ (m)	Wave period T (s)	Angle of wave incidence ϕ_0 (°)	Wave breaking index γ	Particle diameter D_{50} (μm)	Slope of beach profile tgα	Bed roughness r (m)
0.5	4	30	0.8	200	1:100;1:50;1:20	0.06
1.0	5	30	0.8	200	1:100;1:50;1:20	0.06
2.0	7	30	0.8	100;200;300	1:100;1:50;1:20	0.02;0.04;0.06;0.08;0.10
3.0	8	30	0.8	200	1:100;1:50;1:20	0.06
2.0	7	10;20;40;50;60;70;80	0.8	200	1:100;1:50;1:20	0.06
2.0	7	30	0.4;0.6;1.0	200	1:100;1:50;1:20	0.06

TABLE II

Applied bed materials

General indication bed material (D_{50})	Applied in Bijker formula	Applied in Engelund-Hansen formula	Applied in SWANBY and Ackers-White formula
100 μm	D_{50} = 100 μm D_{90} = 175 μm w = .009 m/s ρ_s = 2,650 kg/m^3	D_{50} = 100 μm ρ_s = 2,650 kg/m^3	D_{35} = 85 μm ρ_s = 2,650 kg/m^3 p = 0.31 ν = 10^{-6} m^2/s
200 μm	D_{50} = 200 μm D_{90} = 270 μm w = .025 m/s ρ_s = 2,650 kg/m^3	D_{50} = 200 μm ρ_s = 2,650 kg/m^3	D_{35} = 175 μm ρ_s = 2,650 kg/m^3 p = 0.31 ν = 10^{-6} m^2/s
300 μm	D_{50} = 300 μm D_{90} = 380 μm w = .042 m/s ρ_s = 2,650 kg/m^3	D_{50} = 300 μm ρ_s = 2,650 kg/m^3	D_{35} = 270 μm ρ_s = 2,650 kg/m^3 p = 0.31 ν = 10^{-6} m^2/s

Wave height

In the four transport formulae the mean wave height $\overline{H}$ has been taken as the determining factor in the transport calculations. For computations with the Bijker- and SWANBY formula this seems correct. The results of computations with the adapted Engelund-Hansen and adapted Ackers-White formula are slightly underestimated. The possible error in the ultimate result is very small, since due to the successive wave breaking more and more waves reach the local maximum value γh when the waves are nearing the coast (see Fig. 2B).

4.2 Effect of bed material, beach slope and bed roughness

In Table III the computed sediment transports are summarized as a function of the bed material, the beach slope and the bed roughness. The results of the computations are given in Fig. 5; here the sediment transports as computed with the various formulae have been divided by the calculated CERC transport.

As can be seen from Fig. 5 the four formulae are rather sensitive to the bed material and the bed roughness. With the SWANBY formula and the adapted Ackers-White formula very high transport rates are found when D_{50} = 100 μm is assumed as the bed material diameter. The rates of sediment transport computed with the adapted Ackers-White formula are generally considerably higher than the rates resulting from the SWANBY formula. The Bijker formula is practically insensitive to the bottom slope in contrast to the three other

TABLE III

Sediment transport; effect of bed material, beach slope and bed roughness

D_{50} (μm)	r (m)	S_{Bijker} (m^3/s)			S_{EH} (m^3/s)			S_{SWANBY} (m^3/s)			S_{AW} (m^3/s)		
		tgα			tgα			tgα			tgα		
		1:100	1:50	1:20	1:100	1:50	1:20	1:100	1:50	1:20	1:100	1:50	1:20
100	0.02	.597	.587	.565	.816	1.000	1.990	5.840	25.400	225.000	55.500	89.900	368.000
100	0.04	.305	.299	.280	1.030	1.180	1.940	5.500	23.000	176.000	80.200	111.000	333.000
100	0.06	.197	.196	.183	1.220	1.350	2.000	5.270	21.900	155.000	103.000	132.000	329.000
100	0.08	.142	.143	.133	1.390	1.510	2.080	5.080	21.200	143.000	126.000	154.000	336.000
100	0.10	.109	.110	.104	1.540	1.660	2.180	4.900	20.700	136.000	148.000	176.000	347.000
200	0.02	.248	.265	.322	.408	.502	.994	.065	.255	1.320	.760	1.000	2.260
200	0.04	.184	.188	.202	.517	.591	.972	.042	.170	.890	.765	.929	1.780
200	0.06	.146	.148	.151	.610	.675	.999	.030	.130	.691	.772	.902	1.560
200	0.08	.121	.122	.122	.694	.756	1.040	.024	.105	.571	.779	.890	1.430
200	0.10	.102	.104	.102	.772	.832	1.090	.020	.088	.489	.785	.883	1.350
300	0.02	.113	.125	.172	.272	.334	.663	.020	.075	.323	.242	.299	.553
300	0.04	.106	.110	.128	.345	.394	.648	.011	.046	.209	.218	.254	.419
300	0.06	.096	.099	.107	.407	.450	.666	.008	.033	.158	.205	.233	.356
300	0.08	.087	.089	.093	.463	.504	.694	.005	.025	.127	.196	.219	.318
300	0.10	.079	.081	.083	.514	.555	.727	.004	.020	.106	.189	.208	.291

$H_{0rms} = 2.0$ m; $T = 7$ s; $\phi_0 = 30°$; $\gamma = 0.8$; $S_{CERC} = 0.353$ m^3/s.

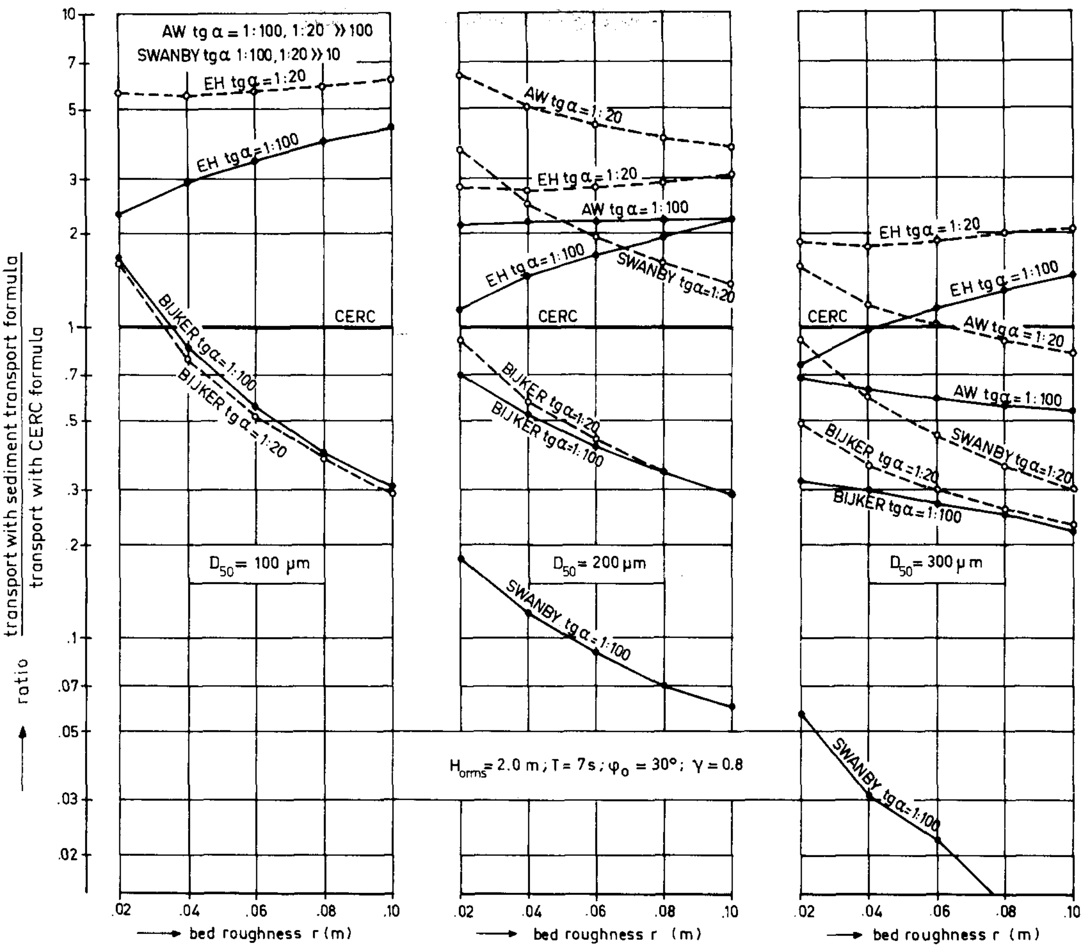

Fig. 5. Effect of bed material and bed roughness.

formulae. In the adopted conception of computing the velocity distribution, the velocity is nearly proportional to the bottom slope. Since the sediment transport computed with the Bijker formula is nearly proportional to the velocity, the final result becomes nearly independent of the slope since with steeper slopes the transport zone is proportionally smaller.

Apart from some attempts made, e.g. by Das (1972) and Swart (1976), to incorporate various effects in the CERC formula, it is generally assumed that bed material, beach slope and bed roughness are not very important in normal prototype conditions. When these assumptions are indeed true, the Bijker formula turns out to be the best one of the four.

4.3 *Effect of wave height*

In Table IV the computations with different wave heights (and periods) have been summarized; Fig. 6 gives a graphical representation. Assuming, as stated before, that the CERC formula describes the longshore transport fairly well, the SWANBY formula seems, with the chosen boundary conditions, rather sensitive to the choice of the wave height. The computations with the adapted Engelund-Hansen and the adapted Ackers-White formula show a slightly better result. The results of the computations with the Bijker formula yield a nearly horizontal line. This points to the same tendency of the Bijker and the CERC formulae.

4.4 *Effect of angle of wave incidence*

The results of the computations with different angles of wave incidence have been listed in Table V; Fig. 7 presents a graph of these results. Fig. 7 shows that the Bijker formula has nearly the same tendency as the CERC formula. The calculations with the Engelund-Hansen, the Ackers-White formula and specially with the SWANBY formula result in a tendency completely different from that of the CERC formula.

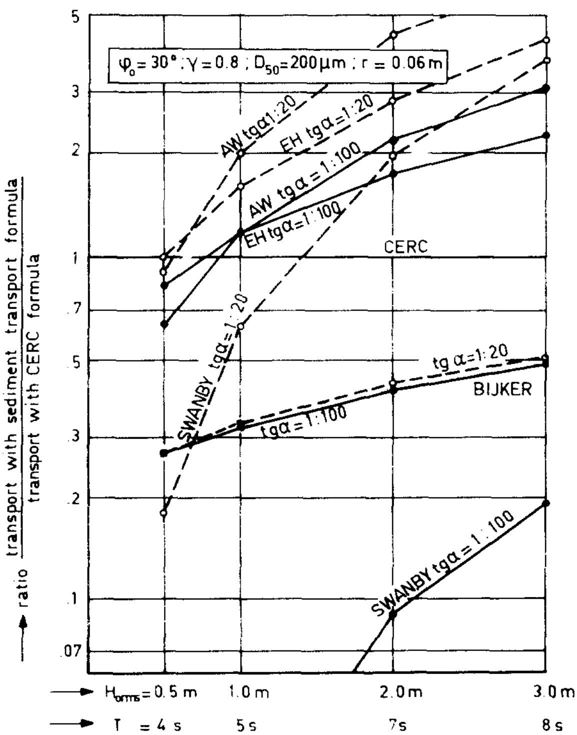

Fig. 6. Effect of wave height.

TABLE IV

Sediment transport; effect of wave height

Wave conditions		S_{CERC} (m^3/s)	S_{Bijker} (m^3/s)			S_{EH} (m^3/s)			S_{SWANBY} (m^3/s)			S_{AW} (m^3/s)		
H_{orms} (m)	T (s)		tgα			tgα			tgα			tgα		
			1:100	1:50	1:20	1:100	1:50	1:20	1:100	1:50	1:20	1:100	1:50	1:20
0.5	4	.011	.003	.003	.003	.009	.010	.011	.000	.000	.002	.007	.008	.010
1.0	5	.063	.020	.021	.021	.074	.080	.101	.001	.006	.040	.074	.083	.124
2.0	7	.353	.146	.148	.151	.610	.675	.999	.030	.130	.691	.772	.902	1.560
3.0	8	.959	.468	.473	.486	2.140	2.450	4.060	.181	.705	3.540	2.960	3.600	6.850

$\phi_0 = 30°$; $\gamma = 0.8$; $D_{50} = 200\ \mu m$; $r = 0.06$ m.

TABLE V

Sediment transport; effect of angle of wave incidence

ϕ_0 (°)	S_{CERC} (m^3/s)	S_{Bijker} (m^3/s)			S_{EH} (m^3/s)			S_{SWANBY} (m^3/s)			S_{AW} (m^3/s)		
		tgα			tgα			tgα			tgα		
		1:100	1:50	1:20	1:100	1:50	1:20	1:100	1:50	1:20	1:100	1:50	1:20
10	.143	.056	.059	.061	.232	.248	.278	.001	.006	.047	.289	.312	.369
20	.266	.110	.113	.114	.455	.490	.630	.011	.050	.291	.571	.634	.923
30	.353	.146	.148	.151	.610	.675	.999	.030	.130	.691	.772	.902	1.560
40	.394	.161	.162	.167	.670	.763	1.250	.050	.202	1.050	.853	1.040	2.020
50	.382	.152	.153	.158	.623	.724	1.250	.057	.225	1.150	.794	1.000	2.070
60	.320	.122	.123	.127	.483	.566	1.000	.046	.183	.940	.612	.781	1.650
70	.221	.078	.079	.082	.291	.338	.584	.025	.101	.528	.361	.457	.952
80	.103	.032	.033	.034	.104	.117	.187	.006	.026	.143	.123	.150	.289

$H_{orms} = 2.0$ m; $T = 7$ s; $\gamma = 0.8$; $D_{50} = 200\ \mu m$; $r = 0.06$ m.

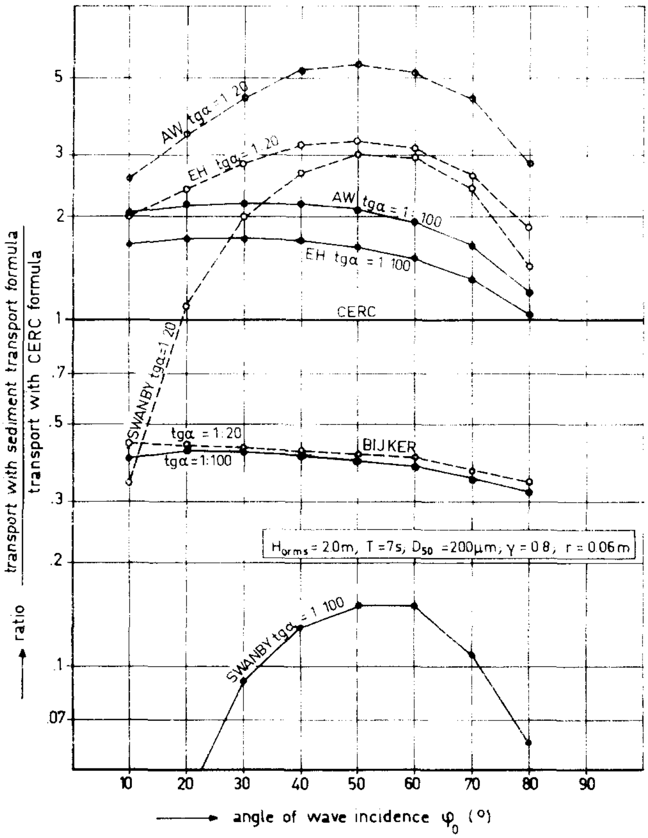

Fig. 7. Effect of angle of wave incidence.

4.5 Effect of wave breaking index

A few computations have been executed with different values of the wave breaking index γ. The results of the calculations are given in Table VI; in Fig. 8 the results are graphically compared with the CERC formula.

The effect of the wave breaking index γ on the CERC transport is indirect. The smaller the γ-value, the further the waves will break from the shoreline. With a small value of γ the refraction phenomenon at the moment of breaking has proceeded less than with a larger γ-value. Hence, $\sin\phi_{br}$ will be relatively larger, and a high transport rate can be expected (see Table VI).

The effect of the wave breaking index is reproduced fairly correctly by the Bijker formula. Contrary to the three other formulae, the Bijker formula predicts, just like the CERC formula, less transport if the γ-value increases.

TABLE VI

Sediment transport; effect of wave breaking index

γ	S_{CERC} (m^3/s)	S_{Bijker} (m^3/s)			S_{EH} (m^3/s)			S_{SWANBY} (m^3/s)			S_{AW} (m^3/s)		
		tgα			tgα			tgα			tgα		
		1:100	1:50	1:20	1:100	1:50	1:20	1:100	1:50	1:20	1:100	1:50	1:20
0.4	.462	.161	.167	.186	.276	.306	.459	.012	.056	.326	.259	.320	.639
0.6	.395	.158	.162	.171	.441	.489	.728	.021	.094	.515	.501	.597	1.090
0.8	.353	.146	.148	.151	.610	.675	.999	.030	.130	.691	.772	.902	1.560
1.0	.324	.133	.134	.134	.783	.864	1.270	.039	.164	.856	1.070	1.230	2.030

H_{0rms} = 2.0 m; T = 7 s; ϕ_0 = 30°; D_{50} = 200 μm; r = 0.06 m.

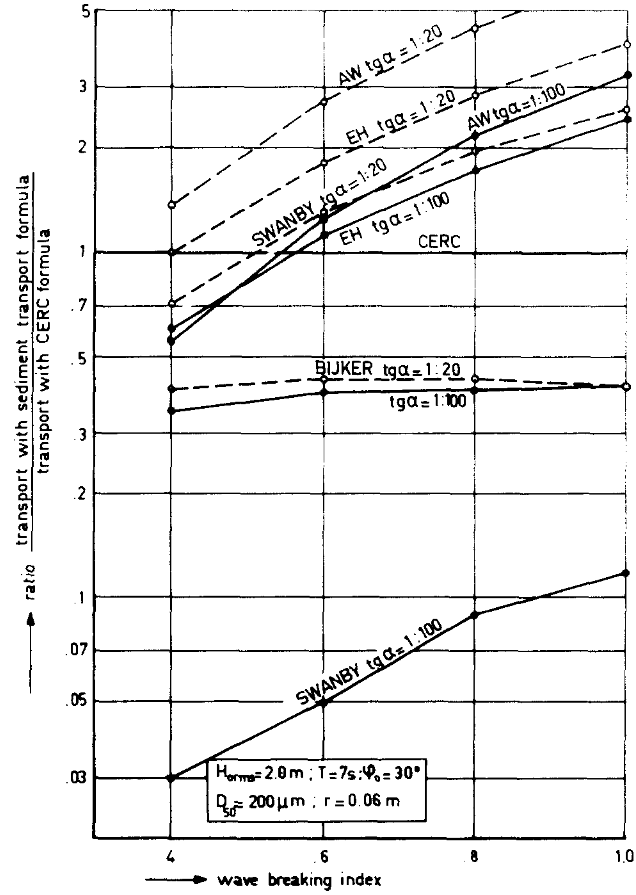

Fig. 8. Effect of wave breaking index.

5. DISCUSSION

Due to the lack of reliable prototype transport measurements in a vertical, a direct verification of the various proposed sediment transport formulae is hardly possible. Hence a preliminary method has been used. The outcome of the Bijker formula (in the ranges tested) is in general closer to the results of the computations with the CERC formula than all other proposed formulae. However, this conclusion should be accepted with the utmost care because there are at least four uncertainties.

(1) The computations with the CERC formula are in fact an unstable basis for comparison. The perpetual discussion around the correct value of the constant of proportionality A proves this convincingly. That is why in weighing the various formulae in the preceding section more attention was

paid to the tendencies compared with those of the CERC formula, than to absolute values.

(2) Before any sediment calculation can be executed using the four formulae, a velocity distribution must be computed. It is well-known, however, that a perfectly reliable distribution can hardly be predicted. Hence five possible distributions have been suggested in paragraph 2.5; this number can be enlarged without any problem. With any chosen velocity distribution, the final result of the comparative calculations depends upon the selected distribution. During the preparation of this paper, computations as made in section 4 have been executed with other velocity distributions. Although the absolute values of the transports did change (sometimes considerably), the tendencies of the ratios between the transports according to the four formulae and the CERC transport point to the same conclusion viz. that the Bijker formula follows the CERC formula closest. To provide impression of the effect of the choice of the velocity distribution on the sediment transport, Table VII shows the total sediment transports which belong to the five distributions as given in Fig. 3.

As can be seen from Table VII the Bijker formula is rather insensitive to the choice of the distribution. Since the Ackers-White, the Engelund-Hansen and especially the SWANBY formula react very strongly on the current velocity, the distributions with relatively strong longshore currents result in a high rate of transport.

(3) The bottom friction concept according to Bijker, as applied in this paper results in a certain velocity distribution. It should be noticed here that other concepts are being developed and that another concept would result in another velocity distribution. Compare for example the work of Bakker (1974) and Bakker et al. (1978).

(4) Under natural prototype conditions, the bottom slope and bed roughness depend in one way or another on the bed particle diameter and the wave conditions. In the present comparative computations the various quantities have nearly always been varied independent of each other. It cannot be

TABLE VII

Effect of choice of velocity distribution

Distribution	S_{Bijker} (m^3/s)	S_{EH} (m^3/s)	S_{SWANBY} (m^3/s)	S_{AW} (m^3/s)
a	.152	1.261	.269	1.846
b	.123	.975	.138	1.400
c	.143	1.015	.081	1.414
d	.159	1.271	.203	1.830
e	.146	.610	.030	.772

$H_{0\mathrm{rms}}$ = 2.0 m; T = 7 s; ϕ_0 = 30°; γ = 0.8; D_{50} = 200 μm; r = 0.06 m; tgα = 1:100; S_{CERC} = 0.353 m^3/s.

excluded, therefore, that some irrelevant combinations have been computed. It is, however, unlikely that the conclusion that the Bijker formula is the better one will have to be rejected if the actual bottom slope and bed roughness belonging to a chosen combination of bed material and wave conditions are introduced.

Neither in this paper nor in the original papers where the adaptation of the Kalinske-Frijlink, the Ackers-White and the Engelund-Hansen formulae is described, is a fundamental justification given for the admissibility of these formulae under coastal-engineering conditions. Undoubtedly the concepts used are not fully trustworthy in every theoretical aspect. Many research activities in detail take place all over the world. However, it is the opinion of the authors that it may take many years before essentially better formulae, which follow physics, will have been developed. It seems therefore acceptable to them to apply imperfect transport formulae in the meantime which have shown more or less reliable results. The Bijker formula seems to be such a formula. A comparison with the results of the CERC formula is only a substitute for actual prototype measurements. A significant part of the investigation efforts should therefore be taken by the acquisition of reliable prototype sediment measurements.

6. CONCLUSIONS

In relatively simple cases the CERC formula can be applied in coastal-engineering practice. In more complicated cases other computation methods should be used. Some concepts have been suggested in the past. The application of these methods is not widespread, because they lack direct verification from prototype and model conditions.

As a first step in the verification of various formulae proposed, a comparison with the results of the CERC formula is suggested. Although the CERC formula is not unobjectionably correct in every respect, this formula has been chosen as a yardstick since it predicts at least a mean value on the basis of quite a number of prototype measurements. Before a comparison of computed transports can be made, a velocity distribution must be calculated. From the various methods proposed in the literature, Battjes' method for irregular waves has been chosen. Various characteristics of the CERC formula (H_0, ϕ_0, γ) have been investigated in the mutual comparison of the proposed formulae. Although this method of comparison has some weaknesses, the conclusion seems to be acceptable that the Bijker formula is a better one than the adapted Ackers-White and adapted Engelund-Hansen formula and a far better one than the adaptation of the Ackers-White formula which is called the SWANBY formula. Furthermore the Bijker formula is, relative to the other formulae, rather insensitive to effects of particle diameter, bottom slope and bed roughness. In using this formula, a possible error in the estimation of the actual conditions results in a slight error in the computed sediment transport.

A similar method of comparison can be used when new computation concepts will be proposed in future.

APPENDIX – NOTATION

a	parameter in error function	—
a, b, c	parameters in bottom friction formula	—
a_0	amplitude of orbital excursion at bed	L
A	dimensionless coefficient in CERC formula	—
A	dimensionless parameter in Ackers-White formula	—
c	wave celerity	$\mathrm{L\,T^{-1}}$
c_g	wave group velocity	$\mathrm{L\,T^{-1}}$
c_0	wave speed in deep water	$\mathrm{L\,T^{-1}}$
C	Chézy friction coefficient	$\mathrm{L^{\frac{1}{2}}T^{-1}}$
C_{90}	idem based on D_{90}	$\mathrm{L^{\frac{1}{2}}T^{-1}}$
C_D	Chézy friction coefficient in Ackers-White formula based on D_{35}	$\mathrm{L^{\frac{1}{2}}T^{-1}}$
$C_{D_{gr}}$	dimensionless parameter in Ackers-White formula	—
D_{35}	particle diameter (65% by weight exceeded in size)	L
D_{50}	particle diameter (50% by weight exceeded in size)	L
D_{90}	particle diameter (10% by weight exceeded in size)	L
D_{gr}	dimensionless grain diameter in Ackers-White formula	—
E	wave energy density	$\mathrm{M\,T^{-2}}$
f	Darcy-Weisbach friction factor	—
f_w	Jonsson's wave friction factor based on r	—
f'_w	idem based on D_{35}	—
F_{wc}	sediment mobility number in Ackers-White formula	—
g	gravitational acceleration	$\mathrm{L\,T^{-2}}$
h	local water depth	L
H	wave height	L
H_f	fictitious wave height	L
H_{max}	maximum wave height	L
H_0	wave height deep water	L
H_{orms}	root mean square wave height deep water	L
H_{sign}	significant wave height	L
I_1, I_2	Einstein's integrals	—
L	characteristic mixing length	L
m	dimensionless parameter in Ackers-White formula	—
M	constant in formula of N_B	—
n	dimensionless parameter in Ackers-White formula	—
n	ratio of group velocity to wave celerity	—
N_B	constant in eddy viscosity formula according to Battjes	—
N_L	idem according to Longuet-Higgins	—
p	porosity	—
p	constant	—

r	bed roughness	L
S	sediment transport	L^2T^{-1}
S_{AW}	(longshore) sediment transport computed with Ackers-White formula	L^2T^{-1} and L^3T^{-1}
S_b	bed load	L^2T^{-1}
S_{Bijker}	(longshore) sediment transport computed with Bijker formula	L^2T^{-1} and L^3T^{-1}
S_{CERC}	longshore sediment transport computed with CERC formula	L^3T^{-1}
S_{EH}	(longshore) sediment transport computed with Engelund-Hansen formula	L^2T^{-1} and L^3T^{-1}
S_{sus}	suspended load	L^2T^{-1}
S_{SWANBY}	(longshore) sediment transport computed with SWANBY formula	L^2T^{-1} and L^3T^{-1}
S_{xx}, S_{yx}, S_{yy}	radiation stress components	$M\,T^{-2}$
t	time	T
T	wave period	T
u	parameter error function	—
u_0	orbital velocity at bed	$L\,T^{-1}$
$\hat{u}_0$	amplitude of the orbital velocity at bed	$L\,T^{-1}$
U	characteristic turbulent velocity	$L\,T^{-1}$
v	mean current velocity	$L\,T^{-1}$
v_{*wc}	shear stress velocity	$L\,T^{-1}$
w	sediment particle fall velocity	$L\,T^{-1}$
x,y,z	coordinates	
α	slope angle	—
γ	wave breaking index	—
Δ	relative apparent density of bed material	—
ϵ	turbulent diffusion coefficient	L^2T^{-1}
κ	Von Kármán coefficient	—
μ	ripple factor	—
ν	kinematic viscosity of water	L^2T^{-1}
ξ	Bijker's parameter based on C and f_w	—
ξ'	idem based on C_D and f'_w	—
π	constant = 3.1416	—
ρ	mass density of water	$M\,L^{-3}$
ρ_s	mass density of bed material	$M\,L^{-3}$
τ'_{wc}	mean bottom shear stress due to waves and current in direction of current	$M\,L^{-1}T^{-2}$
τ_c	bottom shear stress due to current	$M\,L^{-1}T^{-2}$
τ_l	lateral friction force	$M\,T^{-2}$
τ_w	bottom shear stress due to waves	$M\,L^{-1}T^{-2}$
ϕ	angle of wave incidence	—

ϕ_{br}	idem at breaker line	—
ϕ_0	idem in deep water	— ,

REFERENCES

Ackers, P. and White, W.R., 1973. Sediment transport: New approach and analysis. Proc. ASCE, Journal of the Hydraulics Division, HY 11, November 1973.

Anonymous, 1973. Shore Protection Manual. U.S. Army Corps of Engineers, Coastal Engineering Research Center.

Bakker, W.T., 1974. Sand concentration in an oscillatory flow. Proc. Coastal Eng. Conf., 14th, Volume II, chapter 66, Copenhagen.

Bakker, W.T. and Van Doorn, Th., 1978. Near bottom velocities in waves with a current. Paper presented at the 16th Coastal Engineering Conference, Summaries, Paper No. 110, Hamburg.

Battjes, J.A., 1974. Computation of set-up, longshore currents, run-up and overtopping due to wind-generated waves. Delft University of Technology, Communications on Hydraulics, No. 74-2, Department of Civil Engineering.

Battjes, J.A., 1975. Modeling of turbulence in the surf zone. Proceedings of Symposium on Modeling Techniques, ASCE, San Francisco.

Battjes, J.A. and Janssen, J.P.F.M., 1978. Energy loss and set-up due to breaking of random waves. Paper presented at the 16th Coastal Engineering Conference, Summaries, Paper No. 31, Hamburg.

Bijker, E.W., 1967. Some considerations about scales for coastal models with movable bed. Delft Hydraulics Laboratory, Publication No. 50, Delft.

Bijker, E.W., 1971. Longshore transport computations. Proc. ASCE, Journal of the Waterways, Harbors and Coastal Engineering Division, WW4, November 1971.

Bowen, A.J., 1969. The generation of longshore currents on a plane beach. J. Marine Res., 27(2).

Das, M.M., 1972. Suspended sediment and longshore sediment transport data review. Proc. Coastal Eng. Conf., 13th, Volume II, chapter 54, Vancouver.

Einstein, H.A., 1950. The bed-load function for sediment transportation in open channel flows. United States Department of Agriculture, Soil Conservation Service, Technical Bulletin No. 1026.

Engelund, F. and Hansen, E., 1967. A Monograph on Sediment Transport in Alluvial Streams. Teknisk Forlag, Copenhagen.

Frijlink, H.C., 1952. Discussion des formules de débit solide de Kalinske, Einstein et Meyer - Peter et Mueller compte tenue des mesures récentes de transport dans les rivières Néerlandaises. 2me Journal Hydraulique, Soc. Hydraulique de France, Grenoble.

Galvin, C. and Vitale, P., 1976. Longshore transport prediction - S.P.M. 1973 equation. Proc. Coastal Eng. Conf., 15th, Volume II, chapter 67, Honolulu.

Jonsson, I.G., 1966. Wave boundary layers and friction factors. Proc. Coastal Eng. Conf., 10th, Volume I, chapter 10, Tokyo.

Jonsson, I.G., Skovgaard, O. and Jacobsen, T.S., 1974. Computation of longshore currents. Proc. Coastal Eng. Conf., 14th, Volume II, chapter 40, Copenhagen.

Komar, P.D., 1971. The mechanics of sand transport on beaches. J. Geophys. Res., 76.

Longuet-Higgins, M.S., 1970. Longshore currents generated by obliquely incident sea waves. J. Geophys. Res., 75.

Longuet-Higgins, M.S., 1972. Recent progress in the study of longshore currents. In: R.E. Meyer (Editor), Waves on Beaches and Resulting Sediment Transport. Academic Press, New York, London.

Swart, D.H., 1974. Offshore sediment transport and equilibrium beach profiles. Delft Hydraulics Laboratory, Publication No. 131, Delft.
Swart, D.H. and Delft Hydraulics Laboratory, 1976. Coastal sediment transport: computation of longshore transport. Delft Hydraulics Laboratory, Report on Investigation, R 968, part I, Delft.
Swart, D.H., 1976. Predictive equations regarding coastal transports. Proc. Coastal Eng. Conf., 15th, Volume II, chapter 66, Honolulu.
Thornton, E.B., 1970. Variation of longshore current across the surf zone. Proc. Coastal Eng. Conf., 12th, Volume I, chapter 18, Washington.