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Wave height parameter for damage description of rubble-mound breakwaters

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Abstract

In this paper it will be shown that the wave height parameter H_{50}, defined as the average wave height of the 50 highest waves reaching a rubble-mound breakwater in its useful life, can describe the effect of the wave height on the history of the armor damage caused by the wave climate during the structure's usable life.

Using Thompson and Shuttler (Thompson, D.M., Shuttler, R.M., 1975. Riprap design for wind wave attack: A laboratory study on random waves. HRS Wallingford, Report 61, UK) data it will be shown that H_{50} is the wave parameter that best represents the damage evolution with the number of waves in a sea state. Using this H_{50} parameter, formulae as van der Meer (van der Meer, J.W., 1988. Rock slopes and gravel beaches under wave attack. PhD Thesis. Technical University of Delft) and Losada and Giménez-Curto (Losada, M.A., Gimenez–Curto, L.A., 1979. The joint effect of the wave height and period on the stability of rubble mound breakwaters using Iribarren's number. Coastal Engineering, 3, 77–96) are transformed into sea-state damage evolution formulae. Using these H_{50}-transformed formulae for regular and irregular sea states it will be shown how damage predictions are independent of the sea state wave height distribution.

To check the capability of these H_{50}-formulae to predict damage evolution of succession of sea states with different wave height distributions, some stability tests with regular and irregular waves have been carried out. After analysing the experimental results, it will be shown how H_{50}-formulae can predict the observed damage independently of the sea state wave height distribution or the succession of sea states.

Keywords: Rubble-mound; Breakwaters; Stability; Wave height

1. Introduction

The influence of wave height on the armour stability of rubble-mound breakwaters is usually considered by means of $H_{1/n}$, defined as the average wave height of the N/n highest waves of a sea state composed of N waves.

It is well known that for a given $H_{1/n}$ the damage produced by waves on a rubble mound breakwater increases with the duration of the sea state. Consequently, the number of waves of the sea state must be taken into account in the stability formulae as a new parameter. Presently, only van der Meer's (1988) (VdM in the following) formulae take into account the number of waves in the sea state, N.

Many breakwaters are built in intermediate or shallow waters where breaking processes can modify the distribution of wave heights. This modification is especially relevant for the highest waves of the sea state, which are responsible for breakwater damage. Conventional stability formulae are mainly based on experiments carried out for non-breaking conditions and, consequently, they do not properly account for this change of the upper tail of the sea state wave height distribution. Only VdM formulae address this point proposing the use of the $H_{2\%}$ wave instead of H_s. Although this approach improves the prediction of damage, $H_{2\%}$ does not fully takes into account the changes in the wave height distribution that occur in the surf zone and are relevant to breakwater damage. More recently, van Gent et al. (2003), after a series of laboratory stability tests of rubble mound breakwaters on shallow foreshores, proposed using the spectral period $T_{-1,0}$ and re-calibrated VdM formulae to take into account the effect of these changes of wave height distribution due to shoaling and breaking both for mono-modal and bi-modal spectral shapes. The final formulae proposed fits his laboratory tests better than VdM formulae, but it is applicable only for the range of conditions given by the tests because it does not converge to VdM formulae for Rayleigh sea states.

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Damage during the breakwater's usable life will be produced mainly during storms composed of several sea states. Following the same idea, during the life cycle of the breakwater there will be many storms that may produce some damage to the structure. To date, only VdM formulae can take into account the accumulated damage caused for several sea states, using the methodology proposed by van der Meer (1985) to assess the accumulated damage from previous sea states.

From the above paragraphs it is concluded that, in order to adequately predict the damage to a breakwater during its life cycle the following items should be addressed: 1) the number of storms impinging the structure during its life cycle, 2) the number and magnitude of every sea state included in those storms and 3) the duration and wave height distribution of every sea state. It is noted that only VdM formulae addresses all the above points but the distribution of waves in the sea state, which is only partially addressed.

To take into account the aforementioned items, two approaches may be followed: 1) to create a new damage evolution formula or 2) to modify an existing conventional "sea-state formulae" using the appropriate wave parameter so that all the potentially damaging waves are included. Using $H_{2\%}$–VdM formulae only the distribution of waves on the sea states is not fully taken into account.

One example of the first approach can be found in Medina (1996) who proposed an exponential model applicable to individual waves attacking the breakwater and compared the results with the models of Teisson (1990), Smith et al. (1992), and Vidal et al. (1995). Although the method considered the wave period characteristics of individual waves, no experimental contrast was provided.

Following the second approach, Vidal et al. (1995) suggested the utilization of the wave height parameter H_n, defined as the *average wave height of the "n" highest waves that will reach the* breakwater *during its usable life*. This suggestion is based on the fact that at a given time, the damage will be related to the largest waves the breakwater has received up till that instant. In their work, $n=100$ was proposed as a first approximation.

Jensen et al. (1996) carried out a limited number of stability experiments with regular and irregular waves to analyse the suitability of H_n to describe the measured evolution of breakwater damage in a series of consecutive sea states. Only one regular and one irregular test of damage evolution were performed. They concluded that a number of waves of $n=250$ would be appropriate in order to predict the damage evolution and to equalize the damage produced by regular and irregular waves.

In this paper the utilization of the wave height parameter H_n for representing the damage evolution with the number of waves in a sea state is further explored. Using Thompson and Shuttler (1975) data it will be shown that H_{50} is the wave parameter that best represents the damage evolution with the number of waves in a sea state. Using this H_{50} parameter, conventional formulae as VdM and Losada and Gimenez-Curto (1979) (LGC in the following) are transformed into sea-state damage evolution formulae. Using these H_{50}-transformed formulae for regular and irregular sea states it will be shown how damage predictions are independent of the sea state wave height distribution.

To check the capability of these H_{50}-formulae to predict damage evolution of succession of sea states with different wave height distributions, some stability tests with regular and irregular waves have been carried out. After analysing the experimental results, it will be shown how H_{50}-formulae can predict the observed damage independently of the sea state wave height distribution or the succession of sea states, addressing all the aforementioned three weak points of the conventional formulae.

2. Transformation of existing sea state stability formulae using the H_n parameter. Definition of the optimal value for n

VdM formulae for the damage, S, caused by an irregular sea state with significant wave height H_s and mean period T_m composed of N waves, over a rubble mound breakwater with armour stones of size D_{n50}, density ρ_s and relative density $\Delta = \frac{\rho_s}{\rho_w} - 1$ (where ρ_w is the water of density) is given by the following expressions:

$$\frac{H_s}{\Delta D_{n50}} = 6.2 P^{0.18} \left(\frac{S}{\sqrt{N}}\right)^{0.2} \xi_m^{-0.5}; \text{ for } \begin{cases} \xi_m < \xi_{mc} \\ \cot\alpha \leq 4 \end{cases} \text{ and}$$

$$\frac{H_s}{\Delta D_{n50}} = 1.0 P^{-0.13} \left(\frac{S}{\sqrt{N}}\right)^{0.2} \sqrt{\cot\alpha}\, \xi_m^P; \text{ for } \begin{cases} \xi_m \geq \xi_{mc} \\ \cot\alpha \geq 4 \end{cases} \text{ or} \quad (1)$$

with

$$\xi_{mc} = \left(6.2 P^{0.31} \sqrt{\tan\alpha}\right)^{\frac{1}{P+0.5}}$$

where the term $\frac{H_s}{\Delta D_{n50}} = N_s$ in (1) is the stability number and the non-dimensional damage parameter S is defined as the quotient between the average eroded area in the breakwater's sections, A_e and the square of the armour stone size:

$$S = \frac{A_e}{D_{n50}^2} \quad (2)$$

In formulae (1), the surf similarity parameter, ξ_m is defined in terms of the significant wave height, H_s and the mean period, T_m, $\xi_m = \frac{T_m \tan\alpha}{\sqrt{2\pi H_s/g}}$, where $g=9.81\,\mathrm{m/s^2}$ is the gravitational constant.

The permeability parameter P in formulae (1) is defined in terms of the armour, sublayers and core rubble gradations. For example, for a conventional multilayered breakwater with two layers of quarry stone in the armour layer, two layers of quarry stone in the underlayer and a core of quarry run, the value of P should be around 0.4–0.5.

VdM also indicated that the coefficients *6.2* and *1.0* in (1) are normally distributed with mean and standard deviations *0.4* and *0.08*, respectively.

VdM formulae (1) proposed the $(S/\sqrt{N})^{0.2}$ term based on the results of 100 laboratory tests on static stability of rip-rap slopes performed by Thompson and Shuttler (1975) using irregular wave attack. Damage in Thompson and Shuttler (1975) tests was measured after every 1000 waves, up to 5000 waves. Some measurements were also taken after 5000 waves. These tests were reanalysed by VdM in order to show the importance of storm duration on static stability. Tests where the filter layer

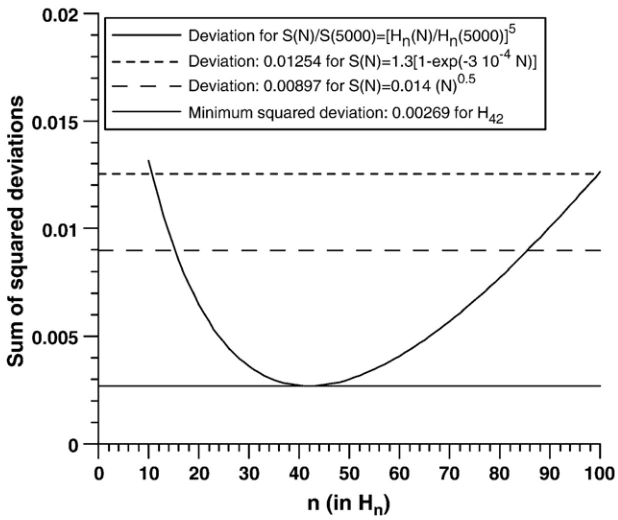


Fig. 1. Sum of squared deviations between Thompson and Shuttler (1975) data for $S(N)/S(5000)$ and expression (7), for different values of n. Also shown are deviations obtained using van der Meer (1988) fits.

became visible after 5000 waves and tests where the damage was very small ($S<2$–3) were omitted. This procedure resulted in a total of about 50 available tests. The value of the damage parameter after N waves, $S(N)$ was related to the damage after 5000 waves, $S(5000)$. Using these data, VdM proposed the following fits for $S(N)/S(5000)$, valid for $N>1000$:

$$\frac{S(N)}{S(5000)} = 1.3\left[1-\exp(3\times 10^{-4}N)\right] \tag{3}$$

$$\frac{S(N)}{S(5000)} = 0.014\sqrt{N} \tag{4}$$

For N between 1000 and 0, VdM proposed to use a linear relationship between the damage after 1000 waves and $S=0$.

VdM also carried out some stability tests and found that the term $\sqrt{N}$ correctly related the damage obtained after 1000 and 3000 waves. For that reason he adopted in formulae (1), the term $S/\sqrt{N}$, that is a constant for a given sea state.

In VdM formulae (1) for a given breakwater geometry and surf similarity parameter, the damage increases with the significant wave height and the number of waves as follows:

$$S = A\sqrt{N}H_s^5 \tag{5}$$

where A is a constant.

If the wave height parameter H_n could take into account the influence of the number of waves on the damage, expression (5) should be written as:

$$S = BH_n^5 \tag{6}$$

and in this case, the relation $S(N)/S(5000)$ should be given directly by:

$$\frac{S(N)}{S(5000)} = \left(\frac{H_n(N)}{H_n(5000)}\right)^5 \tag{7}$$

The optimal value of the number of the sea state's biggest waves n that should be averaged to calculate the H_n parameter has been investigated minimising the squared deviation between the $S(N)/S(5000)$ data of Thompson and Shuttler and the value obtained using expression (7). Taking into account that Thompson and Shuttler (1975) experiments were carried out with the model in intermediate depths without breaking waves, in this analysis the Rayleigh distribution for wave heights in the sea state has been assumed and the $H_n(N)$ parameter of a sea state of N waves has been calculated using Massel (1996) approach:

$$H_n = H_{\frac{n}{N}} = H_{\frac{1}{M}} = \left[\frac{M\sqrt{\pi}}{2}\operatorname{erfc}\left(\sqrt{\ln M}\right) + \sqrt{\ln M}\right]H_{\mathrm{rms}} \tag{8}$$

where $M=N/n$ and erfc is the complementary error function (Abramowitz and Stegun, 1975).

Fig. 1 shows the sum of squared deviations between the data $S(N)/S(5000)$ from Thompson and Shuttler (1975) and the value obtained through H_n using expression (7), for values of n varying between 10 and 100. Also shown in the figure are the corresponding squared deviation between expressions (3) and (4) given by VdM.

From Fig. 1 it can be concluded that any value of n between 16 and 86 fits expression (7) better than expression (4) of VdM. The value of n that produces the minimum deviation to Thompson and Shuttler (1975) data is $n=42$. For the sake of simplicity and taking into account that the statistics of H_n will be less variable as n increases, the value of $n=50$ has been considered optimal.

Fig. 2 shows the fittings to Thompson and Shuttler data obtained using van der Meer (3) and (4) expressions and expression (7) with $n=50$. The three expressions fit the data very well. Expression (7) provides the best fit. Also shown in the figure is the linear relationship proposed for van der Meer for $N<1000$ waves.

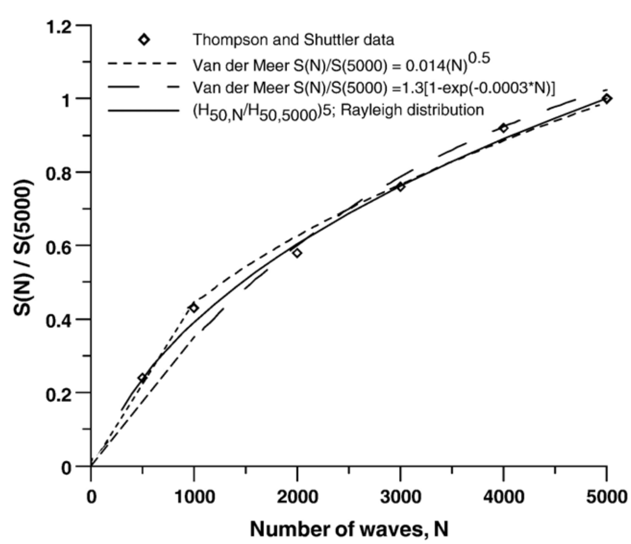


Fig. 2. Comparison between $S(N)/S(5000)$ obtained from Thompson and Shuttler data, van der Meer (1988) fits and expression (7) using $n=50$.

It is worth noting that for a given structure, surf similarity parameter and damage level, the relation between the wave height and the number of waves in VdM formulae (1) is given by the expression:

$$H_s N^{0.1} = AKS^{0.2} \tag{9}$$

where A represents the coefficients 6.2 or 1.0 in VdM formulae and K is a constant.

If $H_{50}(N)$ were used instead of H_s and N, expression (9) should be written as:

$$H_{50} = BKS^{0.2} \tag{10}$$

where B should be a new constant. Dividing Eqs. (9) and (10), the following expression is obtained for the new coefficient:

$$\frac{H_{50}}{H_s N^{0.1}} = \frac{B}{A} = \text{constant.} \tag{11}$$

The value of the B/A constant can be investigated if the distribution of wave height on the sea state is known. Assuming Rayleigh distribution, the value of B/A can be obtained for different number of waves. Calculating the values of B/A for N varying between 500 and 5000 waves, the following mean and standard deviation for B/A values are obtained:

$$\text{Mean of } B/A(N) = 0.716 \tag{12}$$

$$\text{Standard deviation of } B/A(N) = 0.0088 \tag{13}$$

Using the mean value for B/A given by (12), VdM formulae (1) can be transformed to:

$$\frac{H_{50}}{\Delta D_{n50}} = 4.44 P^{0.18} S^{0.2} \xi_m^{-0.5}; \text{ for } \xi_m < \xi_{mc} \text{ and } \cot\alpha \leq 4$$

$$\frac{H_{50}}{\Delta D_{n50}} = 0.716 P^{-0.13} S^{0.2} \sqrt{\cot\alpha}\, \xi_m^P; \text{ for } \xi_m \geq \xi_{mc} \text{ or } \cot\alpha \geq 4$$

with

$$\xi_{mc} = \left(6.2 P^{0.31} \sqrt{\tan\alpha}\right)^{\frac{1}{P+0.5}} \tag{14}$$

Taking into account the low standard deviation of B/A, the new parameters in formulae (14) will again be normally distributed with mean 4.44 and 0.716 and standard deviations 0.286 and 0.057, respectively.

3. Application of H_{50}-formulae to non-Rayleigh sea states

When breakwaters are located in intermediate or shallow waters, maximum waves approaching the breakwaters may start breaking. In these cases, wave height distribution at the breakwater toe is not Rayleigh-distributed. For these cases VdM proposed the substitution of H_s for the wave height that is surpassed by the 2% of the sea state waves, $H_{2\%}$. For a Rayleigh distribution, the relation between H_s and $H_{2\%}$ is 1.4, so VdM transformed his equation (1) substituting H_s by $H_{2\%}/1.4$.

If the distribution of waves is not Rayleigh the value of $H_{2\%}$ do not change gradually as the number of breaking waves increases: $H_{2\%}$ will be the same as in Rayleigh distribution until $H_{2\%}$ breaks. For example: if only 1% of the highest waves break, the value of $H_{2\%}$ will not change compared to the corresponding Rayleigh distribution, but it is clear that the broken waves will reduce the load over the breakwater. The number of broken waves before $H_{2\%}$ differs from the value obtained from Rayleigh distribution depending on the number of waves of the sea state.

For a given breakwater and surf similarity parameter, the relation between the damage predictions of H_{50}-formulae (14) $S_{H_{50}}$, and the conventional VdM formulae (1) S_{H_s}, is expressed by:

$$\frac{S_{H_{50}}}{S_{H_s}} K_1 = \frac{H_{50}}{H_s N^{0.1}} \tag{15}$$

or, if VdM formulae (1) is expressed in terms of $H_{2\%}$:

$$K_2 \frac{S_{H_{50}}}{S_{H_{2\%}}} = \frac{H_{50}}{H_{2\%} N^{0.1}} \tag{16}$$

where K_1 and K_2 are constants.

For Rayleigh-distributed waves, formulae (1) or the version expressed in terms of $H_{2\%}$ and formulae (14) predict similar damage results and the constants K_1 and K_2 will be equal to the value of the right-hand side of Eqs. (15) and (16). This means that the relations between damage predictions (15) and (16) can be written as:

$$\frac{S_{H_{50}}}{S_{H_s}} = \left(\frac{H_{50}}{H_s N^{0.1}}\right)^5 \Big/ \left(\frac{H_{50}}{H_s N^{0.1}}\right)^5_{\text{Rayleigh}} \tag{17}$$

$$\frac{S_{H_{50}}}{S_{H_{2\%}}} = \left(\frac{H_{50}}{H_{2\%} N^{0.1}}\right)^5 \Big/ \left(\frac{H_{50}}{H_{2\%} N^{0.1}}\right)^5_{\text{Rayleigh}} \tag{18}$$

Expressions (17) and (18) could be evaluated using a wave height distribution developed for sea states with broken waves as for example Battjes and Groenendijk (2000), Méndez et al. (2004), but Battjes and Groenendijk distribution require the knowledge of the cero-moment wave height at the structure so a wave propagation model that includes the effect of broken waves should be used and the Méndez et al. distribution is not still thoroughly tested. In this paper a wave-by-wave Montecarlo simulation model, similar to the proposed by Dally (1992) has been used.

The example chosen is a storm with H_s=7 m and T_m=10 s, normally incident to a straight coast with 1/20 slope. The average values of H_s, $H_{2\%}$ and H_{50} parameters and the value of expressions (17) and (18) are obtained at any depth repeating 100 times a wave-by-wave Montecarlo simulation. Simulation results are plot in Fig. 3. In this figure, the horizontal axis corresponds to water depth and the vertical axis to the relations between damage predictions given by expressions (17) and (18).

As can be seen in Fig. 3, the relation between damage predictions for all simulations tends to one (equal prediction of damage) for deep water conditions, where waves are Rayleigh-distributed. As the water depth decreases, these relations become different to one, depending on VdM formulae used or the number of waves simulated.

If the sea state has 360 waves (1 h duration in this case) $S_{H_{50}}/S_{H_s}$ decreases with decreasing water depth. For the extreme case of very shallow water, when the wave height distribution is mostly uniform, H_s–VdM formulae (1) predict damage that is nearly twice the one predicted by H_{50}-formulae (14). For the same case, $H_{2\%}$–VdM formulae predict damage near half the one predicted by H_{50}-formula. These numbers have been obtained using the $\sqrt{N}$ relationship between the damage and the number of waves in VdM formulae for $N>1000$ waves, but the general behaviour do not change too much using the linear relationship. This behaviour changes with the number of waves of the sea state. As the number of waves increases, VdM predictions of S with $H_{2\%}$ become relatively higher with respect to the predictions with H_{50}. For $N=1000$ waves, $H_{2\%}$ and H_{50} are similar for all range of water depths and the predictions of damage with $H_{2\%}$–VdM formulae and H_{50}-formulae (14) are nearly the same. For $N=3000$ waves, $H_{2\%}$ VdM formulae give higher S results than H_{50}-formulae with a maximum deviation for near-regular waves (shallow waters).

As shown in Fig. 3, the maximum deviation between H_s–VdM formulae (1) or their $H_{2\%}$ version and H_{50}-formulae (14) is obtained when waves in the sea state are nearly regular due to the broken waves.

If H_{50}-formulae (14) could predict the damage for regular sea states, their validity for wave height distributions which

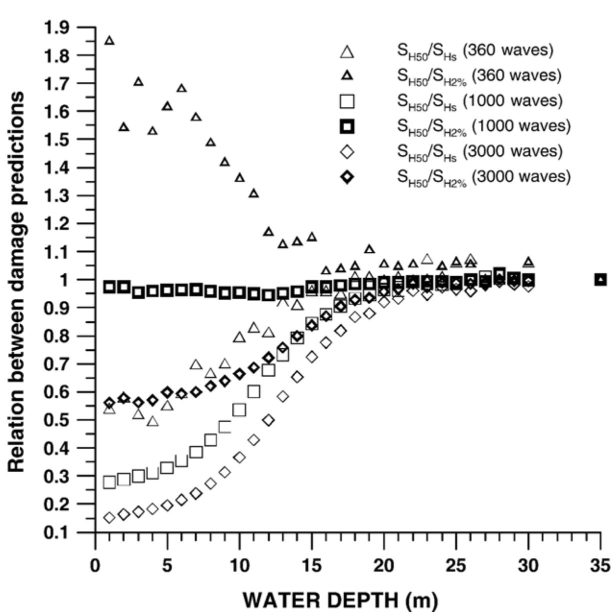


Fig. 3. Montecarlo simulation of the relation between damage predicted by H_{50}-formulae (14) and H_s-van der Meer formulae (1) ($S_{H_{50}}/S_{H_s}$) and between H_{50}-formulae (14) and $H_{2\%}$-van der Meer formulae ($S_{H_{50}}/S_{H_{2\%}}$), for variable water depth and number of waves in the sea state. Deep water wave conditions: $H_s=7$ m, $T_m=10$ s.

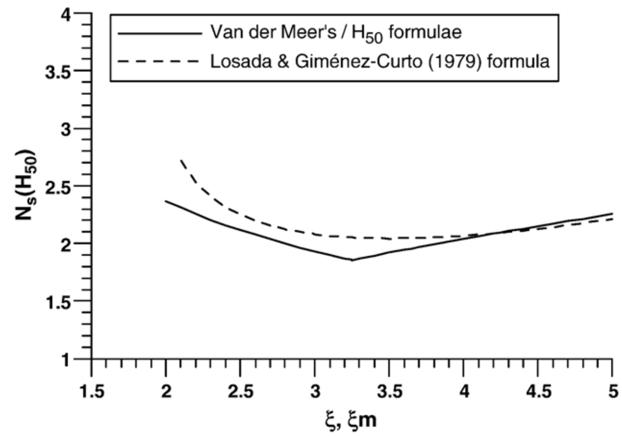


Fig. 4. Variation of the stability number with the surf similarity parameter for a given breakwater and damage level, using H_{50}-formulae (14) and Losada and Gimenez-Curto formulae (19). Case: quarry stones rip-rap armour units, cot$\alpha=2.5$, $S=0.5$ (no damage in Losada and Gimenez-Curto formulae), $A=0.1834$, $B=-0.5764$ and $P=0.45$.

are intermediate between Rayleigh and regular waves will be proven. To do that, first expressions (14) will be used to calculate the stability number for a given damage level of a rubble mound breakwater and compared with results obtained with the experimental formulae of LGC, developed with data from laboratory tests carried out with regular sea states and second, several laboratory tests on damage evolution will be carried out and analysed as explained further in this paper.

LGC best fit formulae for the stability number of rubble-mound breakwater under regular wave attack is given by the expressions:

$$N_s = \left\{ A(\xi-\xi_0)\exp[B(\xi-\xi_0)] \right\}^{-\frac{1}{3}}; \text{ for } \xi > \xi_0 \qquad (19)$$

with

$$\xi_0 = 2.65\tan\alpha$$

where coefficients A and B in (19) depend on the type of armour units, damage level and armour layer slope angle.

Fig. 4 shows a comparison between the stability numbers calculated using H_{50}-formulae (14) and the stability number calculated using LGC formula (19). The case shown in Fig. 4 corresponds to the case of quarry stones rip-rap armour, cot$\alpha=2.5$, $S=0.5$ (no damage in LGC formulae), $A=0.1834$, $B=-0.5764$ and $P=0.45$.

As shown in Fig. 4, the two formulas give similar results, despite their different origin and data base used, confirming the validity of H_{50}-formulae (14) for predicting the damage for regular sea states.

4. Extension of the H_{50} concept to a succession of sea states

The determination of the H_{50} value during the breakwater's usable life can be carried out both analytically or by Montecarlo simulation if (1) the distribution of wave height and periods of the sea states and (2) the significant wave height and period regimes are provided. However, to extend formulae (14) for prediction of damage in a succession of sea states, some assumption about the surf similarity parameter of the succession must be adopted.

The logical period parameter to use to compute the surf similarity parameter in a succession of sea states is T_{50}, the average period of the 50 higher waves in the sea states. The problem is that VdM formulae cannot be re-formulated in terms of T_{50} because the mean period included in the surf similarity parameter does not change during a sea state.

If the highest 50 waves are responsible for the damage, a parameter obtained from the periods of these waves will be convenient to describe the surf similarity parameter. As the sea state with the maximum H_s has a significant contribution to the damage, the use of the surf similarity parameter associated with the significant wave height and mean period corresponding to the sea state of the succession with the maximum significant wave height, $\xi_{mH_{smax}}$ is proposed.

5. Experimental work

In the previous paragraphs it has been demonstrated that the wave parameter H_{50} can take into account the influence of the wave height and number of waves on the damage produced for both Rayleigh and regular sea states on a rubble-mound breakwater. The use of the surf similarity parameter $\xi_{mH_{smax}}$ corresponding to the sea state in the succession with the highest significant wave height has also been suggested.

Now, we will focus on the ability of these parameters to account for the damage evolution on a rubble-mound breakwater submitted to a series of sea states with any wave height and period distribution. First, in order to test the ability of H_{50}-formulae (14) to deal with different sea state statistics, regular and irregular tests on damage evolution were carried out. Next, in order to test the capability of the formulae to predict the damage after a sequence of sea states, different sequences of sea states were tested. It is noted that most of the stability tests were repeated six times to deal with the natural variability on the measured damage in stability tests.

Table 1
Target parameters of regular wave tests with repetition

Test	Serial	H (cm) (1 cm steps)	T (s)	Waves per serial
01	01 to 13	4 to 16	1.00 to 1.20	500
02	01 to 10	6 to 15	1.02 to 1.63	500
03	01 to 08	9 to 16	1.00 to 1.20	500
04	01 to 10	6 to 15	1.02 to 1.63	500
05	01 to 09	8 to 16	1.00 to 1.20	500
06	01 to 12	6 to 17	1.02 to 1.73	500
07	01 to 10	8 to 17	1.00 to 1.24	500
08	01 to 09	6 to 14	1.02 to 1.57	500
09	01 to 08	8 to 15	1.00 to 1.16	500
10	01 to 10	6 to 15	1.02 to 1.63	500
11	01 to 10	8 to 17	1.00 to 1.23	500
12	01 to 11	6 to 16	1.02 to 1.68	500

Table 2
Target parameters of irregular wave tests with test repetition

Test	Serial	H_{mo} (cm) (1 cm steps)	T_p (s)	Waves per serial
13	01 to 08	8 to 15	1.00 to 1.16	1000
14	01 to 06	8 to 13	1.19 to 1.51	1000
15	01 to 09	7 to 15	1.00 to 1.16	1000
16	01 to 07	7 to 13	1.11 to 1.51	1000
17	01 to 10	7 to 16	1.00 to 1.20	1000
18	01 to 07	7 to 13	1.11 to 1.51	1000
19	01 to 10	7 to 16	1.00 to 1.20	1000
20	01 to 08	7 to 14	1.11 to 1.57	1000
21	01 to 08	8 to 15	1.00 to 1.16	1000
22	01 to 06	8 to 13	1.19 to 1.51	1000
23	01 to 09	8 to 16	1.00 to 1.20	1000
24	01 to 08	8 to 15	1.19 to 1.63	1000

5.1. Target wave parameters

A total number of 26 tests of damage evolution were carried out with the target parameters indicated in Tables 1–3.

Three types of tests were carried out:

1— Twelve regular wave tests with two surf similarity parameters ($\xi=2.5$ and $\xi=3.5$) and six repetitions of each test, Tests 01 to 12 in Table 1.
Each test consisted of a series of regular sea states with increasing wave height and period. Each sea state contained 500 waves. At the end of each sea state, damage was measured before the initiation of the next sea state. Each test finished when some of the units of the second layer of the armour were displaced (initiation of destruction). A typical test was comprised of 8 to 13 sea states. The model was rebuilt after each test.
A total of 120 regular sea states (60,000 waves) were carried out.

2— Twelve irregular wave tests with two surf similarity parameters ($\xi_p=2.5$ and $\xi_p=3.5$) and six repetitions of each test, Tests 13 to 24 in Table 2.
Each test consisted of a series of irregular sea states with increasing zero-moment wave height, H_{mo}. Each sea state contained 1000 waves. The rest of the test methodology was as in the regular tests.

Table 3
Target parameters of irregular wave tests with sea state repetition

Test	Serial	Sub-Serial	H_{mo} (cm)	T_p (s)	Waves per sub-serial
D1	1	1 to 5	8	1.00	1000
	2	1 to 5	9	1.00	1000
	3	1 to 5	10	1.00	1000
	4	1 to 5	11	1.00	1000
	5	1 to 5	12	1.04	1000
	6	1 to 5	13	1.08	1000
	7	1 to 5	14	1.12	1000
	8	1 to 5	15	1.16	1000
	9	1 to 5	16	1.20	1000
D2	1	1 to 5	8	1.19	1000
	2	1 to 5	9	1.26	1000
	3	1 to 5	10	1.33	1000
	4	1 to 5	11	1.39	1000

A total of 96 irregular sea states (96,000 waves) were carried out.

3— Two long irregular wave tests with two surf similarity parameters ($\xi_p = 2.5$ and $\xi_p = 3.5$), Tests D1 and D2 in Table 3. Each test consisted of a series of irregular sea estates with increasing zero-moment wave height, H_{mo} and peak period, T_p. Each sea state contained 1000 waves and was repeated five times before increasing wave height and period. The rest of the methodology was as in the previous tests. Test D1 ($\xi_p = 2.5$) required nine wave height stages and a total of 45 sea states (45,000 waves) to reach the prescribed damage. Test D2 ($\xi_p = 3.5$) required four wave height stages and a total of 20 sea states (20,000 waves) to reach the prescribed damage.

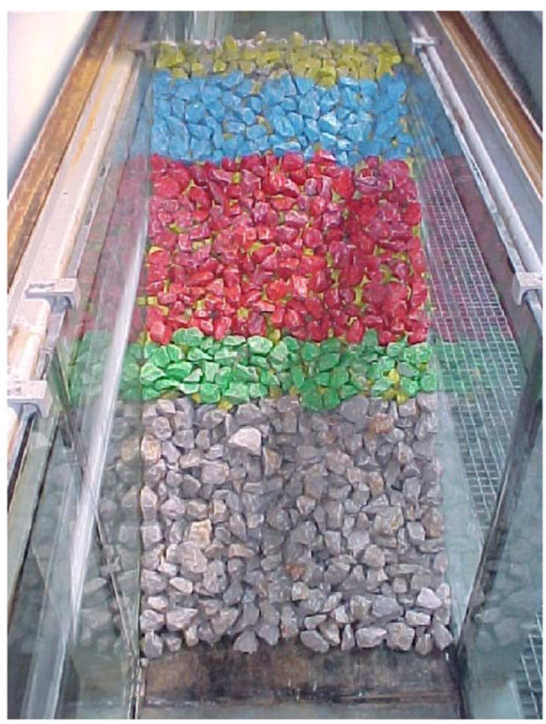
As stated, the aim of the repetition of tests, in Tests 1 to 24, was to take into account the variability of damage due to differences in the armour layer produced by the construction procedure. Long tests D1 and D2 were carried out to check whether the differences on the sequence of sea states affect the damage and whether the H_{50} parameter can take into account those differences.

5.2. Breakwater model characteristics

Tests were carried out at the Coastal Laboratory of the University of Cantabria, Spain. The breakwater model, see photograph and section in Fig. 5, was composed of three types of gravel whose characteristics are described in Table 4. Weight distributions of gravels G0 and G1 were obtained weighing the stones and the size distribution of gravel G2 was obtained through sieving. The outer slope angle was 1/1.5.

To facilitate the assessment of damage, the stones of the two layers of armour rocks were painted with contrasting colours.

(a)



(b)

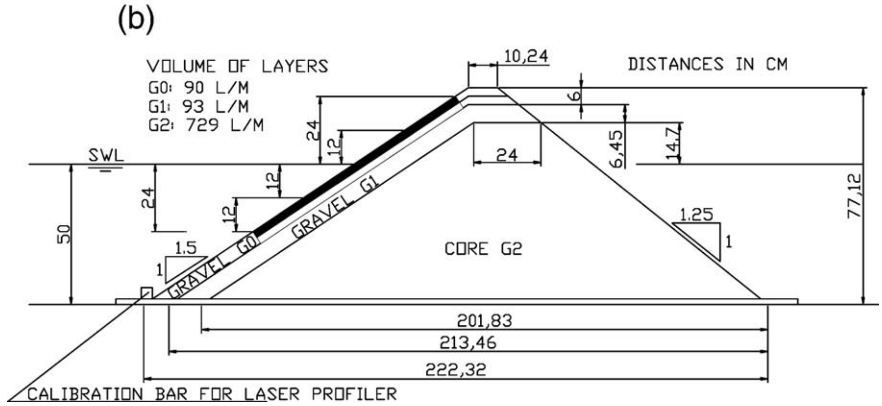


Fig. 5. a) Frontal photograph of the breakwater model and b) model's cross section.

Table 4
Characteristics of gravels in the model

Gravel	$W_{15}\times10^{-3}$, kg	$W_{50}\times10^{-3}$, kg	$W_{85}\times10^{-3}$, kg	$D_{n15}\times10^{-3}$, m	$D_{n50}\times10^{-3}$, m	$D_{n85}\times10^{-3}$, m	Density, kg/m^3	Porosity
G0	47	69	106	*25.9	*29.5	*34.0	2700	0.496
G1	5.6	9.3	14.5	*12.8	*15.1	*17.5	2700	0.490
G2	*0.25	*0.91	*1.80	5.45	6.95	8.73	2700	0.480

Values with * have been obtained through the side of the equivalent cube.

5.3. Experimental set-up

The flume where the model was built, see Fig. 6, is 24 m long and 0.58 m wide and has glass side walls and bottom. The wavemaker is piston-type and it is equipped with two free surface gauges used for the Active Wave Absorption Control System ©AWACS.

On the flume, five free surface gauges were installed: four in the incident side, to separate incident and reflected waves and one on the leeside to measure transmitted waves. A 6-m-long parabolic porous ramp was installed to absorb transmitted energy.

To assess the damage, three methods were used before and after each serial of waves: 1) profiling over 21 cross sections, 2 cm apart, using a laser profiler, 2) counting the removed armour stones settled over the original armour layers and 3) computing the planar eroded area on the outer layer of the armour, using a digital image processing technique. A comparative analysis of these methods of damage assessment can be seen in Vidal et al. (2003). The damage figures presented in this paper are based on method 2.

The number of displaced stones settled over the original armour layers was counted after each serial of waves. Those displaced stones that settled on already eroded areas were not counted. To help in this procedure, digital colour photographs were taken at fixed positions and with fixed optical settings before and after each serial of waves. If the number of counted stones is N_d and the porosity of the settled stones is p, the average eroded area in the width R, A_e, can be obtained from the expression:

$$A_e = \frac{N_d D_{n50}^3}{(1-p)R} \tag{20}$$

where in (20), it has been assumed that the bulk volume of settled stones is the same as the eroded volume. Once the average eroded area was calculated, the damage parameter, S, was computed using the expression (2).

5.4. Wave data analysis

After each sea state the free surface signals from the four gauges located in front of the breakwater were frequency-domain analysed to separate the incident and reflected free surface time series, using the method developed by Baquerizo (1995). Using a zero-downcrossing method, the incident time series was analysed to obtain the incident wave heights and periods. Wave heights were then ordered and compared with the 50 ordered wave heights resulting from previous sea states. From this comparison the new 50 highest waves were stored and the new H_{50} computed. Using this H_{50}, the stability number, N_{s50}, defined by:

$$N_{s50} = \frac{H_{50}}{\Delta D_{n50}} \tag{21}$$

was computed.

6. Analysis of results

Points in Fig. 7 show the stability number, N_{s50} obtained from the tests, in terms of the surf similarity parameter $\xi_{mH_{smax}}$, for a damage level $S=2$. Long irregular tests are represented by squares, regular tests by circles and irregular tests by triangles. Also, H_{50}-formulae (14) and LGC formula (19) with H_{50} and their 5% lower confidence bands are plotted in the figure. The 5% lower confidence band for LGC formulae is obtained multiplying the N_s value given by expression (19) by $(1.41)^{-1/3}$ as recommended by the authors in their paper.

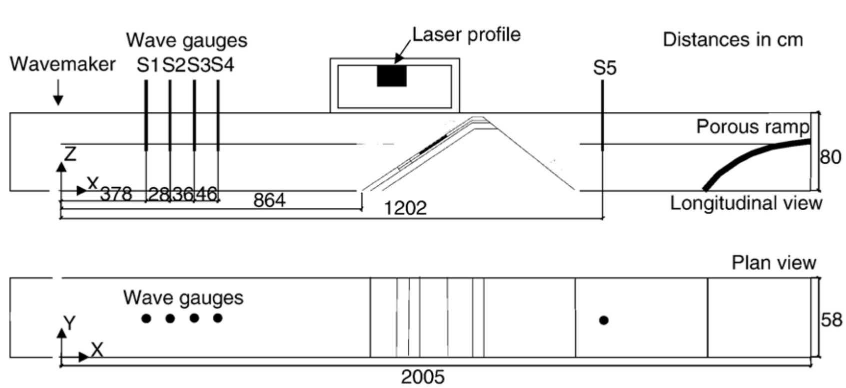


Fig. 6. Experimental set-up.

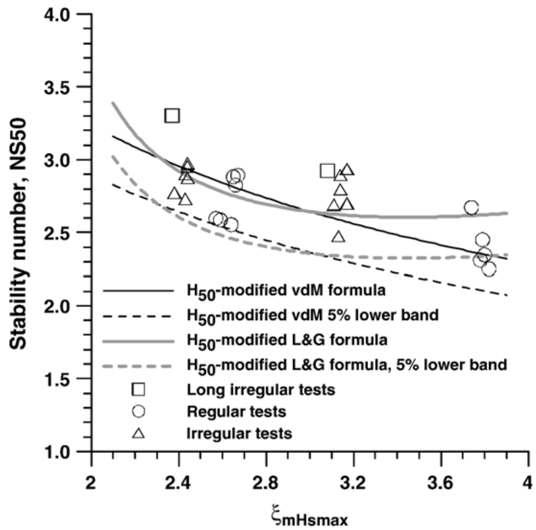


Fig. 7. Test results for N_{s50}, in terms of the surf similarity parameter $\xi_{mH_{smax}}$, for damage level S=2. Solid lines: H_{50}-formulae (14) and Losada and Giménez-Curto's formulae (19). Broken lines: 5% lower confidence bands. Test fixed values: P=0.45, cotα=1.5, Δ=1.7, D_{n50}=0.0295 m, A=0.09035, B=−0.5879, ξ_0=1.75, ξ_{mc}=3.93.

It is worth noting that data results from regular, irregular and long irregular tests are mixed in Fig. 7. The different wave height distributions and sea states sequences are taken into account by the H_{50} parameter in such a manner that the spreading on the N_{s50} and ψ_{50} ($\psi_{50}=1/N_{s50}^3$) parameters coming from different test types is similar to the spreading produced by the repetition of the same test. From the 26 data points, only two (7%) are below LGC's 5% confidence curve and all the points are over VdM 5% confidence band. This means that the proposed H_{50} parameter can take into account the differences in the distribution of the waves on the sea states and the differences in the sequences of sea states.

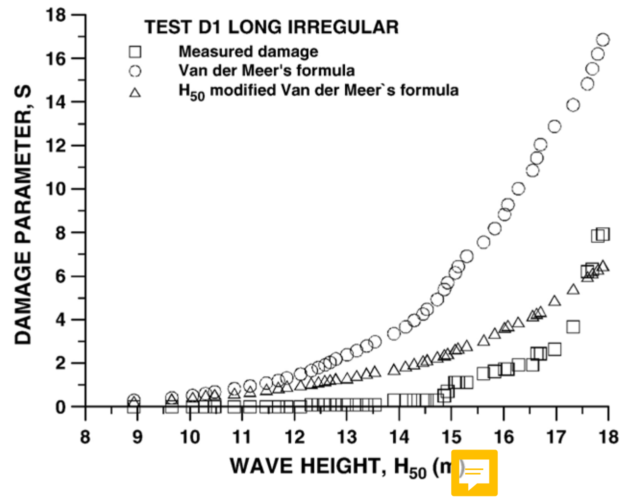


Fig. 9. Measured damage and calculated damage using van der Meer's formulae (1) and H_{50}-formulae (14) for test D1 (long irregular test). Heavy damage (greater than initiation of destruction have been clipped). Test fixed values: P=0.45, cotα=1.5, Δ=1.7, D_{n50}=0.0295 m, ξ_{mc}=3.93.

It should be recalled that these results have been obtained with H_{50}-formulae (14) that were originally developed to assess rubble-mound damage under *only one sea state*, for regular waves (LGC) and for irregular waves (VdM). H_{50}-formulae (14) fit the data correctly because the wave height parameter H_{50} used, allow them to take into account the distribution of waves in a sea state, the number of waves in each sea state and the sea state history of the breakwater.

In Figs. 8–10, a comparison between the measured and predicted damage using VdM formulae (1) and H_{50}-formulae (14) is presented for three selected tests: short irregular test D14 (Fig. 8) long irregular test D1 (Fig. 9) and short regular test D02 (Fig. 10). In all figures, the horizontal axis represents the measured H_{50} and the vertical axis shows the damage

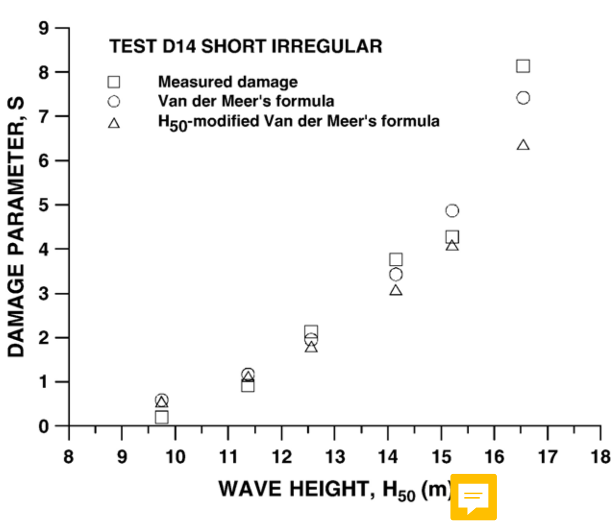


Fig. 8. Measured damage and calculated damage using van der Meer's formulae (1) and H_{50}-formulae (14) for test D14 (short irregular test). Heavy damage (greater than initiation of destruction have been clipped). Test fixed values: P=0.45, cotα=1.5, Δ=1.7, D_{n50}=0.0295 m, ξ_{mc}=3.93.

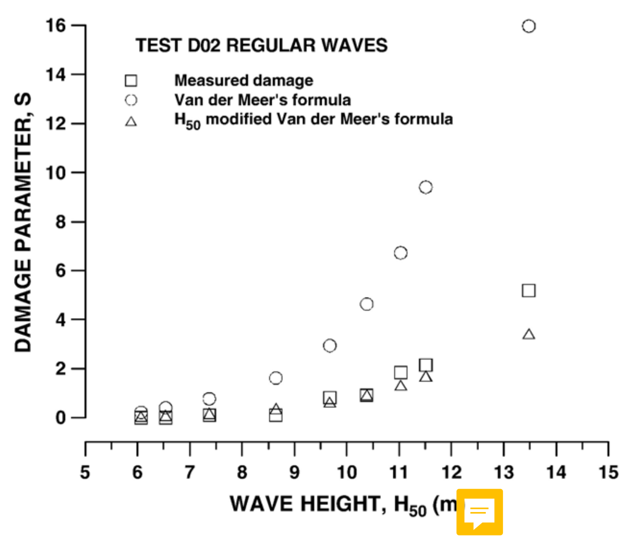


Fig. 10. Measured damage and calculated damage using van der Meer's formulae (1) and H_{50}-formulae (14) for test D02 (short regular test). Heavy damage (greater than initiation of destruction have been clipped out). Test fixed values: P=0.45, cotα=1.5, Δ=1.7, D_{n50}=0.0295 m, ξ_{mc}=3.93.

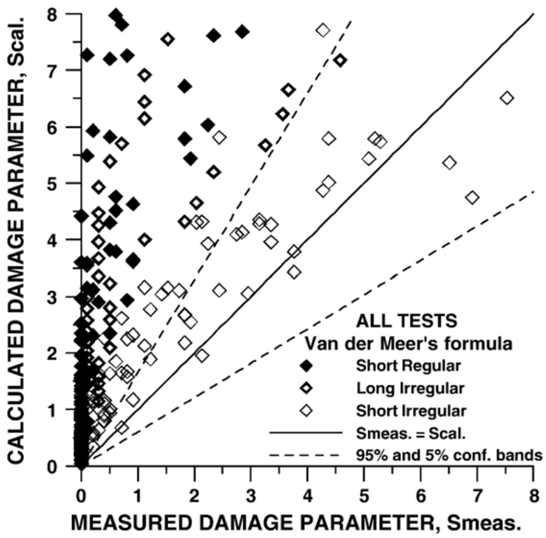


Fig. 11. Measured and calculated damage parameter using van der Meer's formulae (1) and his methodology, van der Meer (1985). All test with measured damage $S<8$. Test fixed values: $P=0.45$, $\cot\alpha=1.5$, $\Delta=1.7$, $D_{n50}=0.0295$ m, $\xi_{mc}=3.93$.

parameter, S. Damage with S > 8, corresponding to damage in the second layer of the armour, are not shown in the figures.

To assess the evolution of damage in a succession of sea states using VdM formulae (1), the methodology recommended by van der Meer (1985) has been used:

1. For the first sea state of the sequence, composed of N waves, the damage parameter is calculated using formulae (1) and the measured significant wave height and surf similarity parameter.
2. For the rest of sea states, the number of waves that produce the damage calculated in the previous sea state is computed using formulae (1) with the present significant wave height and surf similarity parameter. These "previous" waves are added to the number of waves of the present sea state and the final damage is calculated using formulae (1).

When a sequence of Rayleigh-distributed sea states of increasing wave height are used in the test, Fig. 8, both, VdM formulae (1) and H_{50}-formulae (14) give a good approach to the measured damage.

Fig. 9 shows the comparison between the measured and predicted damage for the long irregular test D1. In this test, each sea state is repeated exactly five times before a new increase of the significant wave height. As a result, the distribution of wave height after the sequence of five equal sea states is not Rayleigh and the maximum waves after 5000 waves are smaller than those predicted using Rayleigh (in fact they are the maximum waves of a train of 1000 waves repeated 5 times). As VdM formulae assume Rayleigh distribution of the waves, it overestimates the damage and the evolution curve goes clearly over the measured evolution. As can be seen on the figure, H_{50}-formulae, despite overestimating S in the first stages of damage in this test, predicts much better the evolution of damage.

Fig. 10 shows the comparison between the measured and predicted damage for the regular test D02. Regular waves are one extreme case of non-Rayleigh distribution. Using the significant wave height in VdM formulae (that was not developed for use with regular waves), the predicted damage should be much greater than that measured because the formulae assumes that the maximum waves are Rayleigh-distributed. On the other hand, the parameter H_{50}, equal in this case to the regular wave height, takes into account the real size of the maximum waves and the formulae (14) predicted damage evolution fits very well the measured one.

Finally, Figs. 11 and 12 show a comparison between measured and predicted damage for all tests, using VdM formulae (1) Fig. 11, and using H_{50}-formulae (14) Fig. 12. Also shown in the figures are the two 5% and 95% confidence bands for Eqs. (1) and (14). Again, heavy damage over S = 8 (initiation of destruction) are not shown.

Fig. 11 shows that VdM's formulae (1) is unable to properly predict the evolution of damage in series of sea states with different wave height distributions. Although a clear overestimation of damage under regular sea states is expected, the accumulation of damage carried out by the exact repetition of irregular sea states (long irregular tests) is also clearly overestimated, with all the points but one being outside the 95% confidence band. The evolution of damage in a succession of Rayleigh sea states is, however, well predicted by VdM formulae and methodology. As VdM formulae was developed using damage values $S>1.5$ it is logical that they overestimates the evolution of damage for small damage values ($S<2$) and works much better for higher damage values.

In Fig. 12, H_{50}-formulae (14) is used to compare measured and predicted damage, for all tests with $S<8$. From this figure, the following points can be drawn:

– Long irregular tests, regular and standard irregular tests are mixed in the plot, indicating that H_{50}-formulae (14) can

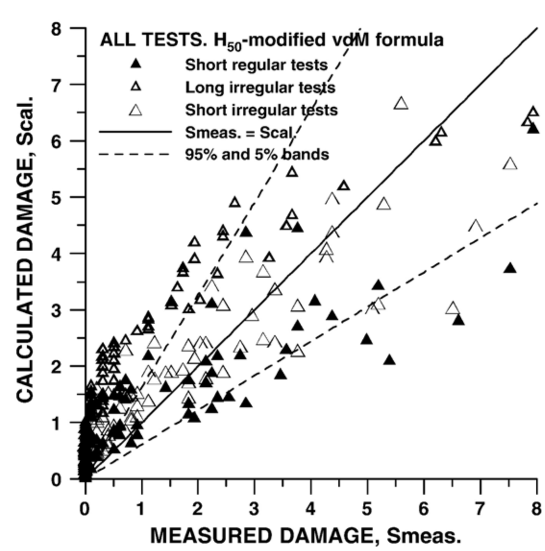


Fig. 12. Measured and calculated damage parameter using H_{50}-formulae (14). All tests with measured damage $S<8$. Test fixed values: $P=0.45$, $\cot\alpha=1.5$, $\Delta=1.7$, $D_{n50}=0.0295$ m, $\xi_{mc}=3.93$.

describe the damage evolution for a series of sea states with different wave height and period distributions.

– For small damage values, $S<1.2\,H_{50}$-formulae (14) overpredicts the damage.
– For high values of damage, $S>5$, H_{50}-formulae (14) slightly underpredict the damage.
– For damage between 1 and 8, the evolution of damage for Rayleigh-distributed sea states is very well predicted by H_{50}-formulae (14).
– For the long irregular tests (one type non-Rayleigh irregular sea states) the damage for $S<2.5$ is overestimated. This overestimation disappears for values of $S>2.5$.

The evolution of damage under series of regular sea states shows a higher dispersion than for the irregular sea states, but, on average, the points are well spread around the measured values.

7. Conclusions and recommendations

In this paper it has been shown that the wave height parameter H_{50}, defined as the average wave height of the 50 highest waves reaching a rubble-mound breakwater in its useful life, can describe the effect of the wave height on the history of the armor damage caused by the wave climate during the structure's usable life.

Using Thompson and Shuttler (1975) data, it has been demonstrated that the H_{50} parameter can describe the damage on Rayleigh-distributed sea states of any length as well as the present existing formulae that take into account the number of waves in the sea states, such as VdM.

It has been shown that the current sea-state stability formulae for rubble-mound breakwaters can be easily modified to make use of the wave height parameter H_{50}. In this paper, VdM (1) and LGC (19) formulae have been chosen as example. In H_{50}-formulae (14) the influence of the number of waves is taken by the parameter H_{50}. Using H_{50}-formulae, the influence of the wave height distribution on the damage disappears and both aforementioned formulae can be used to calculate the armour stability for sea states having wave height distribution as different as Rayleigh or monochromatic. This means that H_{50}-formulae are capable of describing the damage to the armour of rubble mound breakwaters located in shallow or intermediate water depths, where wave height distribution during storms can depart from Rayleigh due to non-linearities or breaking processes.

Using laboratory tests with sea states having different wave height distributions it has been demonstrated that the only present stability formulae capable of describing the evolution of damage in a succession of sea states, VdM formulae with the methodology developed by van der Meer (1985), can only describe properly the evolution of damage in near Rayleigh-distributed successions of sea states. It also has been demonstrated that H_{50}-formulae (14) can be used to calculate the evolution of the damage during different sequences of sea states having any wave height distribution and duration, i.e, they are capable of describing the damage during the structure's useful life, making them applicable to probabilistic analysis of failure.

The calculus of H_{50} requires detailed information of the incident wave statistics at the structure's toe, both for short term (sea states) and long term (wave regimes). Once these statistics are known, H_{50} can be calculated analytically or by Montecarlo simulation. As in time-limited sea states, H_{50} is a random variable, the use of the mean H_{50} is recommended when using multiple Montecarlo simulations of the structure's useful life.

The advantage of the H_{50} approach for potential designers is a better description of damage in the case of shallow waters and the capacity of the H_{50} parameter to make use of the stability data base developed with regular wave tests as is the case of LGC formulae.

In the case of laboratory experiments on rubble-mound armour stability, the application of H_{50}-formulae requires that all the incident wave heights and periods at the structure toe must be calculated from the free surface measurements.

List of symbols

A, B, K, K_1, K_2 Constants and coefficients.
A_e Average eroded area in the breakwater's section.
D_{n15} Side of the equivalent cube of a stone which weight is not surpassed by 15% of the weight in the weight distribution curve.
D_{n50} Side of the equivalent cube of a stone which weight is not surpassed by 50% of the weight in the weight distribution curve.
D_{n85} Side of the equivalent cube of a stone which weight is not surpassed by 85% of the weight in the weight distribution curve.
$g=9.81\,m/s^2$ Gravity acceleration.
H_n Average wave height of the n highest waves reaching a breakwater in its useful life.
H_{50} Average wave height of the 50 highest waves reaching a breakwater in its useful life.
$H_{1/n}$ Average wave height of the N/n highest waves of a sea state composed of N waves.
H_{m0} Cero-moment wave height.
H_s or $H_{1/3}$ Significant wave height or average wave height of the N/3 highest waves of a sea state composed by N waves.
$H_{1/10}$ Average wave height of the N/10 highest waves of a sea state composed of N waves.
$H_{2\%}$ Wave height surpassed by the 2% highest waves in a sea state.
H_{smax} Maximum significant wave height of a succession of sea states.
L_0 Deep water wave length for regular waves.
L_{0m} Deep water wave length associated to the mean period of a sea state.
M N/n
N Number of waves of a sea state.
N_d Number of displaced stones forming more than two layers on the breakwater slope.
$N_s=H_s/(\Delta D_{n50})$ Stability parameter defined with H_s.
$N_{s50}=H_{50}/(\Delta D_{n50})$ Stability parameter defined with H_{50}.

P Breakwater notional porosity parameter of van der Meer.
p Armour porosity.
R Length of the breakwater where the displaced stones are counted.
$S=A_e/D_{2n50}$ Damage parameter.
$S_{H_{50}}$ Damage prediction using H_{50}.
S_{H_s} Damage prediction using H_s.
T_m Average wave period in a sea state.
T_p Peak period of a sea state.
W_{15} Stone weight not surpassed by 15% of the weight in the weight distribution curve.
W_{50} Stone weight not surpassed by 50% of the weight in the weight distribution curve.
W_{85} Stone weight not surpassed by 85% of the weight in the weight distribution curve.
α Breakwater's armour slope angle.
$\xi=\tan\alpha/\sqrt{(H/L_0)}$ Surf similarity parameter for regular waves.
ξ_p Surf similarity parameter associated to H_s and T_p.
$\xi_m=\tan\alpha/\sqrt{(H_s/L_{0m})}$ Surf similarity parameter for irregular waves in a sea state.
ξ_{mc} Critical surf similarity parameter that separates the field of application of the two van der Meer formulae.
ξ_0 Surf similarity parameter for Losada and Gimenez-Curto (1979) formulae.
$\xi_{mH_{smax}}$ Surf similarity parameter associated with $H_{2\%}$ and T_m corresponding to the sea state with the maximum H_{smax} of a succession of sea states.
ρ_s Density of the armour stones.
ρ_w Density of the water.
$\Delta=(\rho_s/\rho_w)-1$ Relative density of the armour stones.

Acknowledgements

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